

Collegio Carlo Alberto



Disaster recovery and the term structure of dividend strips

Michael Hasler

Roberto Marfè

No. 458

July 2016

Carlo Alberto Notebooks

www.carloalberto.org/research/working-papers

Disaster recovery and the term structure of dividend strips^{*}

Michael Hasler^{a*}, Roberto Marfè^b

Journal of Financial Economics, forthcoming

^aUniversity of Toronto, 105 St. George Street, Toronto, ON, M5S 3E6, Canada.

^bCollegio Carlo Alberto, Via Real Collegio, 30, 10024 Moncalieri (Torino), Italy.

Abstract

Recent empirical findings document downward-sloping term structures of equity return volatility and risk premia. An equilibrium model with rare disasters followed by recoveries helps reconcile theory with empirical observations. Indeed, recoveries outweigh the upward-sloping effect of time-varying disaster intensity and expected growth, generating downward-sloping term structures of dividend growth risk, equity return volatility, and equity risk premia. In addition, the term structure of interest rates is upward-sloping when accounting for recoveries and downward-sloping otherwise. The model quantitatively reconciles high risk premia and a low risk-free rate with the shape of the term structures, which are at odds in other models.

JEL classification: D53; D51; E21; G12; G13

Keywords: Recovery; Rare disasters; Term structures of equity; Dividend strips; Asset pricing puzzles

*We would like to thank Ilaria Piatti, European Finance Association (EFA) meeting 2015 discussant, G. William Schwert (the editor), and an anonymous referee for comments that significantly improved the quality of the paper. We also would like to acknowledge comments from Giovanna Nicodano, Julien Penasse, and conference participants at the EFA meeting 2015. Financial support from the University of Toronto, the Connaught New Researcher Award, and the Collegio Carlo Alberto is gratefully acknowledge.

*Corresponding author. Tel.: +1 (416) 946-4014.
Email address: michael.hasler@rotman.utoronto.ca

1. Introduction

Recent empirical studies show that the term structures of dividend growth risk, equity return volatility, and equity risk premia are downward-sloping (van Binsbergen, Brandt, and Koijen, 2012; van Binsbergen, Hueskes, Koijen, and Vrugt, 2013).¹ These findings are particularly important because they question the validity of the most successful asset pricing models. Models featuring time-varying disaster risk (Gabaix, 2012; Wachter, 2013) or time-varying expected growth (Bansal and Yaron, 2004) provide convincing foundations for the observed properties of equity return volatility and risk premia, but they assume term structures of dividend growth volatility and, in turn, imply term structures of equity return volatility and risk premia that are inconsistent with the data.

In this paper, we account for the empirically supported fact that dividends recover after disasters (Gourio, 2008; Nakamura, Steinsson, Barro, and Ursua, 2013). While either natural or man-made disasters affect both physical and, to a lesser extent, human capital, disaster recovery is easy to understand by means of knowledge conservation. Available technology and know-how allow accelerated post-disaster economic growth. Capital accumulation is easier the second time around because it replicates a known investment pattern. Moreover, disasters induce government spending to stimulate the economic environment and foster competition.²

We show that disaster recovery helps explain the observed shape of the term structures of equity return. We provide theoretical evidence that recoveries kill the upward-sloping effect of both time-varying disaster risk and time-varying expected growth. The reason is that, in the presence of recoveries, the volatility of dividend growth is larger in the short term than in the long term. In equilibrium, the properties of dividend growth volatility transmit to

¹See van Binsbergen and Koijen (2016) for a recent survey on the term structures of equity return.

²While debate is ongoing about the short-run and long-run impacts of disasters on economic growth, developed economies, such as the US, seem to be able to mitigate adverse effects and exploit growth opportunities. See Cavallo, Galiani, Noy, and Pantano (2013) and references therein.

stock returns and imply empirically consistent downward-sloping term structures of equity return volatility and risk premia.

We base our model on three important properties of the joint dynamics of consumption and dividends for which we provide evidence. First, dividends scaled by consumption are stationary, i.e. consumption and dividends are co-integrated (Lettau and Ludvigson, 2005). Second, the term structure of consumption growth volatility is slightly upward-sloping, and that of dividend growth volatility is markedly downward-sloping (Marfè, 2016). This heterogeneity in the timing of fundamental risk is key to understanding the shape of the term structures of equity return. Third, conditional on a disaster, dividends drop more and recover faster than consumption (Longstaff and Piazzesi, 2004), validating the theoretical predictions of Gourio (2012).

These stylized facts are closely related to each other. Co-integration implies that consumption and dividends face the same permanent shock and that the dividend share of consumption is stationary. However, the dividend share of consumption moves negatively with disasters and positively with recoveries. This implies that disasters are (at least partially) transitory shocks and that dividends drop more and recover faster than consumption. The levered exposition of dividends to transitory risk helps explain the gap between the upward-sloping term structure of consumption growth risk and the downward-sloping term structure of dividend growth risk.

We consider a pure-exchange economy (Lucas, 1978) with a representative investor who has Epstein and Zin (1989) preferences. To model the joint dynamics of consumption and dividends, we consider a rare disaster model with time-varying disaster intensity (Wachter, 2013) and expected growth (Bansal and Yaron, 2004). The key feature of our model is that recoveries take place right after the occurrence of a disaster (Gourio, 2008). We argue that, even though disaster intensity risk and long-run risk imply empirically inconsistent term structures of equity return, extending these models with plausible recoveries helps explain

simultaneously several important properties of dividends, consumption, and asset prices.³

We show that the presence of recoveries implies a markedly downward-sloping term structure of dividend growth volatility. The reason is that, in the short term (e.g., one day), the dividend faces disaster and expected growth risk, but the horizon is too short to benefit from a significant recovery. In the long term (e.g., 20 years), disaster and expected growth risk are still present, but dividends have a significant amount of time to recover. However, the speed of consumption recovery is not high enough to outweigh disaster intensity risk and expected growth risk, both of which imply a large amount of risk in the long term. Therefore, the term structure of consumption growth volatility is slightly upward-sloping.

In equilibrium, equity returns inherit the properties of dividend growth rates. Therefore, the term structure of equity return volatility and risk premia are, as the term structure of dividend growth volatility, downward-sloping in our model. To understand the dynamic patterns of the term structures of equity return over the business cycle, we define bad (good) economic times as states of the world in which the disaster intensity is high (low). Consistent with van Binsbergen et al. (2013), we show that the slopes of the term structures of equity return are pro-cyclical in our model, being smaller in bad times than in good times. The reason is that, in bad times, the disaster intensity is large and is consequently expected to revert back down to its mean in the long term. This implies a significantly larger amount of risk in the short term than in the long term and, therefore, steep downward-sloping term structures of equity return. In good times, however, the disaster intensity is small and eventually reverts back up to its long-term mean. That is, disaster intensity risk is larger in the long term than in the short term. This dampens the downward-sloping effect of recoveries and implies flatter term structures of equity return. The properties of the term structures of equity return hold even for an elasticity of intertemporal substitution (EIS) smaller than

³Consistent with the international evidence presented by Gourio (2008) and Nakamura et al. (2013), our calibration implies that consumption partially recovers after large drops (by about 50%) in about five years. This is the joint result of long-run growth and after-disaster excessive conditional growth.

one.⁴ The reason is that returns inherit the properties of cash flow growth rates as long as the EIS is larger than some lower bound. This lower bound turns out to be smaller than one for equity –the dividend claim– because dividends have a levered exposition on disaster risk.

In line with empirical findings, the risk-free rate is about 0.6% and equity risk premia are about 6.5%. The model generates relatively large risk premia because, in the presence of recoveries, risk premia decrease with the elasticity of intertemporal substitution (Gourio, 2008).⁵ Moreover, the ability of the model to solve the risk-free rate and equity premium puzzle and simultaneously capture the negative slope of the term structures of equity return is robust to the setting of investors' preferences. We show that our results hold for an elasticity of intertemporal substitution in the range (0.5,2) and for a coefficient of relative risk aversion in the range (4.4,6.6).

Furthermore, we show that recoveries help explain the observed shape of the term structure of real interest rates. The term structure of interest rates is upward-sloping in the presence of recoveries, and it is downward-sloping when disasters are permanent. The reason is that the term structure of consumption growth risk is significantly less upward-sloping in the presence of recoveries. Because bonds are used to hedge consumption growth risk, bond yields increase with maturity when there are recoveries and decrease with it when disasters are permanent. Finally, the model implies an inverse relation between the current price-dividend ratio and future excess returns, in line with the empirical findings of Cochrane (2008) and van Binsbergen and Kojien (2010). The predictive power, however, decreases with the forecasting horizon because predictability comes mainly from transitory sources of risk in our model.

Our model builds on the literature about rare disasters (Tsai and Wachter, 2015). Models

⁴A major critique of long-run risk and rare disasters models is their reliance on a large elasticity of intertemporal substitution (Epstein, Farhi, and Strzalecki, 2014).

⁵If disasters are fully permanent, then risk premia increase with the elasticity of intertemporal substitution.

featuring time-varying risk of disasters provide a theoretical explanation of a number of price patterns (Gabaix, 2012; Wachter, 2013) and have found empirical support (as in Berkman, Jacobsen, and Lee (2011), among others). We complement this literature by pointing out the importance of recovery (Gourio, 2008) and focusing on the otherwise puzzling term structures of both fundamentals growth and equity return.

Because our results rely on the existence of recoveries, our paper is closely related to the findings of Nakamura et al. (2013) who fit a rare disasters model to international consumption data and provide evidence that consumption disasters unfold over a few years and that consumption recovers by about 50% in the following five years. They show that the presence of recoveries significantly decreases equilibrium risk premia on the consumption claim and that reasonably large risk premia can be obtained only if the elasticity of intertemporal substitution is large. In contrast with their study, in our model the dynamics of consumption and dividends are heterogeneous and we focus on the impact of recoveries on the term structures of consumption growth risk, dividend growth risk, equity return volatility and risk premia, and interest rates. We show that, because dividends are more sensitive to disasters and recoveries than consumption, the shape of each of the five term structures is consistent with empirical findings.

A few recent papers investigate the properties of the term structures of dividend growth risk and equity return. Ai, Croce, Diercks, and Li (2012), Belo, Collin-Dufresne, and Goldstein (2015), and Marfè (2014) focus, respectively, on investment risk, financial leverage, and labor relations. These macroeconomic channels contribute to endogenize the downward-sloping term structure of dividend growth risk and, in turn, of equity return. This paper differs from these studies because it focuses on an endowment economy in which dividends feature time-varying disaster risk and recover after the occurrence of a disaster. The model calibration provides support to the idea that the disaster-recovery channel has a potentially sizable quantitative impact on asset price dynamics. Other theoretical studies that help explain the shape of the term structures of equity return focus on non standard specifications

of beliefs formation and preferences. These are proposed by Croce, Lettau, and Ludvigson (2015) and Berrada, Detemple, and Rindisbacher (2013), respectively. In addition, Lettau and Wachter (2007, 2011) show in a partial equilibrium setting that a pricing kernel which enhances short-run risk can help to simultaneously explain the shape of the term structures of equity return and the cross-sectional value premium. Similar to Marfè (2014), our equilibrium framework allows us to endogenize the pricing kernel and to provide an economic rationale for the intuition provided in Lettau and Wachter (2007, 2011). As in Longstaff and Piazzesi (2004), we assume that the dividend share of consumption is stationary. While they focus on the relation between the dynamics of corporate cash flows and equity risk premia, we are concerned with the impact of recoveries and recursive preferences on the term structures of equity return.

The remainder of the paper is organized as follows. Section 2 provides empirical support to the main assumptions and economic mechanism. Section 3 presents and solves the model. Section 4 describes the term structures of consumption growth risk, dividend growth risk, equity return volatility and risk premia, and interest rates. Section 5 presents the term structures of equity return when disasters unfold over a few years and Section 6 concludes. Derivations are provided in the Appendix.

2. Empirical support

Gourio (2008) provides evidence that rare disasters are followed by recoveries. This means that, after large drops, gross domestic product and consumption growth feature a conditional mean that is larger than the unconditional one. Such evidence suggests that, in contrast with what is usually considered in the literature, rare disasters should be modeled, at least partially, as transitory shocks instead of permanent shocks. More recently, Nakamura, Steinsson, Barro, and Ursua (2013) provide international evidence that disasters unfold over a few years and then partially recover. Whether disasters have a transitory or permanent

nature and whether they take place slowly or rapidly are key to the extent of modeling the term structure of risk of macroeconomic fundamentals, i.e., the volatility of growth rates computed over different time horizons or the corresponding variance ratios.

Belo, Collin-Dufresne, and Goldstein (2015) and Marfè (2014, 2016) show that aggregate dividend growth is characterized by a markedly downward-sloping term structure of risk and that aggregate consumption growth features a slightly upward-sloping term structure of risk. The corresponding variance ratios are reported in Fig. 1. The different shape of the two term structures can be interpreted as follows. Assume that both consumption and dividends are driven by a permanent and a transitory shock (Lettau and Ludvigson, 2014). These shocks can feature time-varying expected growth, volatility or jump intensity. Variations in the conditional distribution of the permanent shock induce an upward-sloping effect, and variations in the conditional distribution of the transitory shock induce a downward-sloping effect. The combination of these two effects determines the shape of the term structures. As long as consumption and dividends are co-integrated (Lettau and Ludvigson, 2005), they share the same permanent shock but can be exposed differently to the transitory shock. In fact, such a heterogeneity in the exposure to transitory risk explains the difference between the shape of the term structure of consumption growth risk and that of dividend growth risk. Consumption loads to a lesser extent on the transitory shock than dividends do. Therefore, the upward-sloping effect dominates in the case of consumption and the term structure of consumption growth risk increases with the horizon. In contrast, the downward-sloping effect dominates in the case of dividends and the term structure of dividend growth risk decreases with the horizon. An implication of this mechanism is that the dividend share of consumption is stationary and increases with the transitory shock.

[Insert Fig. 1 about here.]

In what follows we provide empirical support to the previous interpretation and relate it to the role of disasters and recoveries. Panel A of Table 1 reports summary statistics

about real consumption and dividends. Data are from the National Income and Products Accounts (NIPA) tables 1.1.6, 1.1.10, and 1.11 and concern the aggregate US economy on the sample 1929–2013 at yearly frequency. Consumption and dividend growth rates share a similar unconditional mean, but the former is smoother than the latter. Consumption growth is positively autocorrelated, and dividend growth is negatively autocorrelated. In addition, consumption and dividend growth rates are positively but imperfectly correlated. The excess volatility of dividend growth rates, their negative autocorrelation, and their imperfect correlation with consumption growth rates are three stylized facts consistent with the idea that dividends load more than consumption on a transitory shock. The dividend share is small on average, smooth, relatively persistent, and negatively correlated with the one-period ahead growth rates of consumption and dividends. This is also consistent with the idea that the dividend share is increasing with the transitory component of consumption and dividends.

[Insert Table 1 about here.]

In Panel B, we perform a Johansen test of co-integration in a vector error correction model (VECM). Both the Schwarz Bayesian (SBIC) and the Hannan and Quinn (HQIC) information criteria suggest that consumption and dividends are co-integrated. We should reject the null hypothesis of no co-integrating equations, and we cannot reject the null hypothesis of one co-integrating equation.⁶

Consistent with the property of co-integration, Panel C shows that the dividend share is stationary. The coefficient obtained by regressing the change in the dividend share on its lagged value is negative and significant (the Newey-West t-statistics is -3.74). In accord with our interpretation, co-integration implies that the negative slope of dividend growth variance ratios is due to the excessive exposition of dividends to transitory risk.

⁶This result is robust to the lag and trend specifications and also holds when using postwar data only.

The upper graphic of Fig. 2 shows the time series of the logarithm of aggregate dividends and highlights the rare disaster events defined as growth rates smaller than two standard deviations below the average. Consistent with Gourio (2008) and Nakamura, Steinsson, Barro, and Ursua (2013), we observe that a substantial recovery occurs during the years following the rare events. The middle graphic of Fig. 2 shows the time series of the dividend share of consumption. We observe that the dividend share moves negatively with disasters and positively with recovery. We show this more formally in Panel D of Table 1. We construct a disaster-period dummy (Dis) that is equal to one during years in which growth is smaller than two standard deviations below the average as well as during the subsequent negative growth years. In addition, we construct a recovery dummy (Rec) that is equal to one the year following a disaster. Because the dividend share is stationary, it cannot depend on the permanent component of consumption and dividends. Therefore, the fact that coefficients associated to the dummy variables Dis and Rec are statistically significant provides evidence that the event of a disaster followed by a recovery is (at least partially) transitory. Moreover, the coefficients are respectively negative and positive, consistent with the idea that dividends drop more and recover faster than consumption when a disaster occurs.⁷

[Insert Fig. 2 about here.]

The lower graphic of Fig. 2 shows the time series of the (standardized) changes in the dividend share and the predicted time series obtained by regressing the changes in the dividend share on the dummy variables Dis and Rec. The joint explanatory power of the dummy variables Dis and Rec is large (adjusted $R^2 = 46\%$), meaning that the disaster/recovery path is

⁷Many asset pricing models, such as Wachter (2013) and Martin (2013), among others, assume that dividends load more on disasters than consumption does. This is consistent with the general equilibrium model of Gourio (2012) in which dividends are endogenous and with the empirical findings of Longstaff and Piazzesi (2004): “While aggregate consumption declined nearly 10% during the early stages of the Great Depression, aggregate corporate earnings were completely obliterated, falling more than 103%.”

a main driver of the dividend share dynamics. In turn, given the stationarity of the dividend share, the disaster/recovery path is a main driver of the transitory component of consumption and dividends. This helps explain why dividend growth is riskier than consumption growth at short horizons and why the term structures of dividend and consumption growth risk are downward-sloping and upward-sloping, respectively.

Overall, we observe seven stylized facts: (1) the timing of dividend growth risk is markedly downward-sloping; (2) the timing of consumption growth risk is slightly upward-sloping; (3) dividend growth is riskier than consumption growth at short horizons; (4) dividends and consumption are co-integrated; (5) disasters are followed by recoveries; (6) dividends load more on disaster risk than consumption does, and (7) the dividend share moves with disasters and recoveries.

In Section 3, we present a model that captures the seven stylized facts as well as the partial recovery property shown by Nakamura, Steinsson, Barro, and Ursua (2013). In Section 4, we show that the model can simultaneously explain several observed properties of asset prices such as the downward-sloping term structures of equity return volatility and risk premia, the pro-cyclical dynamics of their slopes, the upward-sloping term structure of interest rates, the low risk-free rate, and the high equity risk premia.

3. A model of rare disasters and recoveries

In this section, we first describe the economy and characterize equilibrium asset prices. Then, we calibrate the dynamics of aggregate consumption and dividends to capture the empirical patterns shown in Section 2.

3.1. The economy

We consider a pure-exchange economy à la Lucas (1978) populated by a representative investor with recursive preferences (Epstein and Zin, 1989). The investor's utility function

is defined by

$$U_t \equiv \left[(1 - \delta^{dt}) C_t^{\frac{1-\gamma}{\theta}} + \delta^{dt} \mathbb{E}_t (U_{t+dt}^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \quad (1)$$

where C_t is consumption, δ is the subjective discount factor per unit of time, γ is the coefficient of risk aversion, ψ is the elasticity of intertemporal substitution (EIS), and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$.

Aggregate consumption, C_t , and the dividend paid by equity, D_t , are characterized as

$$\log C_t = x_t + z_{1t} \quad (2)$$

and

$$\log D_t = x_t + z_{2t} + \log d_0, \quad d_0 \in (0, 1), \quad (3)$$

with

$$dx_t = (\mu_t - \frac{1}{2}\sigma_x^2 + \omega z_{1t})dt + \sigma_x dW_{xt}, \quad (4)$$

$$dz_{1t} = -\phi_z z_{1t} dt + \xi_t dN_{zt}, \quad (5)$$

$$dz_{2t} = -\phi_z z_{2t} dt + \alpha \xi_t dN_{zt} + \sigma_z dW_{zt}, \quad (6)$$

$$d\lambda_t = \phi_\lambda (\bar{\lambda} - \lambda_t) dt + \sigma_\lambda \sqrt{\lambda_t} dW_{\lambda t}, \quad (7)$$

and

$$d\mu_t = \phi_\mu (\bar{\mu} - \mu_t) dt + \sigma_\mu dW_{\mu t}, \quad (8)$$

where $(W_{xt}, W_{zt}, W_{\lambda t}, W_{\mu t})^\top$ is a standard Brownian motion and N_{zt} is a Poisson process with stochastic intensity λ_t . The jump size ξ_t follows a negative time invariant exponential

distribution with parameter η . That is, the jump size is negative and characterized by the moment-generating function

$$\varrho(u) \equiv \mathbb{E}_t(e^{u\xi_t}) = \frac{1}{1 + \frac{u}{\eta}}. \quad (9)$$

The log-consumption and log-dividend processes consist of two components. The first, x_t , is the growth rate had there been no disasters and consequently no recoveries either. The aim of the second component, z_{it} , $i \in \{1, 2\}$, is to model disasters and recoveries. Conditional on the occurrence of a disaster ($dN_{zt} = 1$), the log-consumption and log-dividend drop instantaneously by an amount ξ_t and $\alpha\xi_t$, respectively. Following the drop, the process z_{it} reverts back to zero at speed ϕ_z and, therefore, implies a recovery in both consumption and dividends. If the mean-reversion speed ϕ_z is equal to zero, there are no recoveries and disasters are permanent.⁸ The above dynamics show that if $\alpha = d_0 = 1$ and $\sigma_z = 0$, then consumption is equal to dividends.

Because the permanent shock x_t is a common driver of $\log C_t$ and $\log D_t$, consumption and dividends are co-integrated in levels, as observed in the data. To obtain an upward-sloping term structure of consumption growth volatility and realistic equity risk premia, we consider time variation in expected growth μ_t (Bansal and Yaron, 2004). As long as $\phi_z > 0$, a disaster is followed by a recovery, but the recovery is only partial (Nakamura, Steinsson, Barro, and Ursua, 2013) because disasters induce a permanent drop through the expected growth of x_t ($\omega > 0$). In addition, the model embeds a levered exposition of dividends to disasters ($\alpha > 1$). This assumption produces three properties: (1) although dividends and consumption are co-integrated, dividend growth is more volatile than consumption growth at short horizons; (2) the term structure of dividend growth risk is downward-sloping because $\alpha > 1$ allows the

⁸Nakamura, Steinsson, Barro, and Ursua (2013) point out that disasters are not necessarily instantaneous. We keep the model simple here and do not account for “slow” disasters because they do not alter the main qualitative result of the paper, that is, recoveries imply downward-sloping term structures of equity return. However, we model “slow” disasters and discuss their implications in Section 5.

downward-sloping effect implied by recoveries to outweigh the upward-sloping effect implied by long-run risk and time-varying intensity; and (3) the dividend share $D_t/C_t = d_0 e^{z_2 t - z_1 t}$ decreases with disasters and increases with recoveries.⁹ The noise parameter σ_z is added to the dynamics of dividends because it helps to simultaneously match the consumption growth volatility, the dividend growth volatility, and the dividend share volatility observed in the data.¹⁰

3.2. *Equilibrium*

To solve for the prices of dividend strips, we follow the methodology presented by Eraker and Shaliastovich (2008), which is based on the Campbell and Shiller (1988) log linearization. The first step consists of characterizing the price of the claim to aggregate consumption, the state-price density, and, therefore, the risk-neutral measure. Then, the price at time t of a dividend strip with time-to-maturity τ is obtained by computing the expected present value under the risk-neutral measure of a dividend $D_{t+\tau}$ paid at time $t + \tau$.

Recursive preferences lead to a non-affine state-price density. Therefore, we make use of the following log-linearization to preserve analytic tractability. The discrete time (continuously compounded) log-return on aggregate wealth V (e.g., the claim on the representative investor's consumption) can be expressed as

$$\log R_{t+1} = \log \frac{V_{t+1} + C_{t+1}}{V_t} = \log (e^{v_{c,t+1}} + 1) - v_{c,t} + \log \frac{C_{t+1}}{C_t}, \quad (10)$$

where $v_{c,t} = \log(V_t/C_t)$. A log-linearization of the first summand around the mean log

⁹Provided that $\alpha > 1$, this statement holds even though the speed of recovery ϕ_z is the same for both log-consumption and log-dividend.

¹⁰Even with recursive utility, the risk associated with σ_z is not priced. This modeling assumption is intended to make unambiguous the relation between disasters and recoveries and equity risk premia.

wealth-consumption ratio leads to

$$\log R_{t+1} \approx k_0 + k_1 v_{c,t+1} - v_{c,t} + \log \frac{C_{t+1}}{C_t}, \quad (11)$$

where the endogenous constants k_0 and k_1 satisfy

$$k_0 = -\log \left((1 - k_1)^{1-k_1} k_1^{k_1} \right) \quad (12)$$

and

$$k_1 = e^{\mathbb{E}(v_{c,t})} / (1 + e^{\mathbb{E}(v_{c,t})}). \quad (13)$$

In a Gaussian environment, Campbell, Lo, and MacKinlay (1997) and Bansal, Kiku, and Yaron (2012) show the high accuracy of such a log-linearization, which we assume exact hereafter.¹¹ We follow Eraker and Shaliastovich (2008) and consider the continuous time counterpart defined as:

$$d \log R_t = k_0 dt + k_1 dv_{c,t} - (1 - k_1)v_{c,t} dt + d \log C_t. \quad (14)$$

Recursive preferences lead to the following Euler equation, which enables us to characterize the state-price density, M_t , used to price any asset in the economy:

$$\mathbb{E}_t \left[\exp \left(\log \frac{M_{t+\tau}}{M_t} + \int_t^{t+\tau} d \log R_s \right) \right] = 1. \quad (15)$$

¹¹In Appendix E, we show that the log-linearization remains accurate when the model accounts for disasters (i.e., jumps in consumption). We perform a log-linearization of the wealth return dynamics only. We do not need to perform a second log-linearization when computing either dividend strip prices or the price of equity. Therefore, the log-linearization affects prices only through the intertemporal component of the state-price density, which is a function of wealth under recursive utility. Moreover, we show in Section 4 that dividend strip returns inherit the term-structure properties of dividend growth rates. That is, our results are mostly implied by the cash flow channel as opposed to the discount rate channel. In turn, the quantitative impact of the log-linearization on our results is likely to be negligible.

The state-price density satisfies

$$d \log M_t = \theta \log \delta dt - \frac{\theta}{\psi} d \log C_t - (1 - \theta) d \log R_t. \quad (16)$$

To solve for the return on aggregate wealth and, in turn, on the state-price density, one has to conjecture that $v_{c,t}$ is affine in the vector of state variables $Y_t = (x_t, z_{1t}, z_{2t}, \lambda_t, \mu_t)^\top$. Then, the Euler equation is used to solve for the coefficients. The wealth of the representative investor is characterized in Proposition 1.

Proposition 1. *The investor's wealth, V_t , satisfies*

$$v_{c,t} \equiv \log \frac{V_t}{C_t} = A_c + B_c^\top Y_t, \quad (17)$$

where \top is the transpose operator and $Y_t = (x_t, z_{1t}, z_{2t}, \lambda_t, \mu_t)^\top$ is the vector of state variables. The state variables belong to the affine class and their dynamics can be written as

$$dY_t = \mu(Y_t)dt + \Sigma(Y_t)dW_t + J_t \cdot dN_t, \quad (18)$$

$$\mu(Y_t) = \mathcal{M} + \mathcal{K}Y_t, \quad (19)$$

$$\Sigma(Y_t)\Sigma(Y_t)^\top = h + \sum_{i=1}^5 H^i Y_t^i, \quad (20)$$

and

$$l(Y_t) = l_0 + l_1 Y_t, \quad (21)$$

where $\mathcal{M} \in \mathbb{R}^5$, $\mathcal{K} \in \mathbb{R}^{5 \times 5}$, $h \in \mathbb{R}^{5 \times 5}$, $H \in \mathbb{R}^{5 \times 5 \times 5}$, $l_0 \in \mathbb{R}^5$, and $l_1 \in \mathbb{R}^{5 \times 5}$, $W_t = (W_{x_t}, W_{z_{1t}}, W_{z_{2t}}, W_{\lambda_t}, W_{\mu_t})^\top$ is a standard Brownian motion; $N_t = (N_{x_t}, N_{z_{1t}}, N_{z_{2t}}, N_{\lambda_t}, N_{\mu_t})^\top$ is a vector of independent pure jump processes; $l(Y_t) \in \mathbb{R}^5$ is the corresponding vector of jump intensities; $J_t \in \mathbb{R}^5$ is the vector of jump sizes; and \cdot denotes element-by-element multiplication. The coefficients $A_c \in \mathbb{R}$ and $B_c \in \mathbb{R}^5$ solve a system of equations provided in

Appendix A.

Proof. See Appendix A.

The wealth-consumption ratio is stationary in our model and consequently independent of x_t ($B_{c,1} = 0$). Moreover, the wealth-consumption ratio decreases (increases) with the intensity λ_t when the elasticity of intertemporal substitution is larger (smaller) than one. The reason is that an increase in the intensity increases the likelihood that negative jumps in consumption will take place in the future. This has two opposite effects on the wealth-consumption ratio. First, the possibility of more frequent downward jumps implies lower expected future consumption. Because the wealth-consumption ratio is the (scaled) expectation of discounted future consumption, an increase in the intensity tends to decrease the wealth-consumption ratio through a substitution effect. Second, lower expected future consumption pushes the investor to save more today for future consumption purposes. This income effect tends to increase the wealth-consumption ratio. The substitution effect dominates when the elasticity of intertemporal substitution is larger than one and the income effect is the strongest when the elasticity is smaller than one.

For a similar reason, the wealth-consumption ratio decreases (increases) with the jump process z_{1t} when the EIS is larger (smaller) than one. A drop in z_{1t} , today, indicates that future consumption growth will be large because a recovery will take place. Because the substitution effect dominates when the EIS is larger than one, a drop in z_{1t} implies an increase in the wealth-consumption ratio. Conversely, a drop in z_{1t} implies a decrease in the wealth-consumption ratio when the EIS is smaller than one because, in this case, the income effect dominates.

The risk-free rate r_t , the market price of continuous risk Λ_t^c , and the market price of jump risk Λ_t^d are defined in Proposition 2.

Proposition 2. *The dynamics of the state-price density, M_t , are written*

$$\frac{dM_t}{M_t} = -r_t dt - \Lambda_t^c dW_t - \sum_{i=1}^5 \left(\Lambda_t^{d^i} dN_t^i - \mathbb{E}_t \left(\Lambda_t^{d^i} \right) l^i(Y_t) dt \right), \quad (22)$$

where the risk-free rate r_t , the market price of continuous risk Λ_t^c , and the market price of jump risk Λ_t^d satisfy

$$r_t = \Phi_0 + \Phi_1^\top Y_t, \quad (23)$$

$$\Lambda_t^c = \Sigma(Y_t)^\top \Omega, \quad (24)$$

and

$$\Lambda_t^{d^i} = 1 - e^{-\Omega^i J_t^i}, \quad \forall i. \quad (25)$$

The coefficient $\Phi_0 \in \mathbb{R}$ and $\Phi_1 \in \mathbb{R}^5$ solve a system of equations provided in Appendix B.

Proof. See Appendix B.

The risk-free rate is stationary and therefore independent of x_t . It is, however, a decreasing linear function of the intensity λ_t , irrespective of the value assigned to the EIS. The reason is that an increase in the disaster intensity yields an increase in consumption growth risk, which the investor is willing to hedge with risk-free investments. This willingness to increase risk-free holdings implies an increase in the price of the risk-free asset and thus a decrease in the risk-free rate. Such an effect increases in magnitude with relative risk aversion. The decreasing linear relation between the risk-free rate and the disaster process z_{1t} is understood as follows. A disaster, i.e., a drop in z_{1t} today, is followed by a recovery and therefore implies high expected future consumption growth. This decreases the investor's willingness to hedge using the risk-free asset and, hence, increases the risk-free rate.

The first component of the market price of continuous risk Λ_t^c is constant and equal to $\gamma\sigma_x$ (Lucas, 1978). It rewards the investor for bearing the constant dividend growth risk.

The second component of Λ_t^c is associated with the jump intensity. This term is stochastic because the jump intensity follows a square-root process (see Eq. (7)). The last component of Λ_t^c is associated with time variation in expected growth. It is positive (negative) under preference for early (late) resolution of uncertainty (Bansal and Yaron, 2004). Moreover, the investor gets a reward Λ_t^d for bearing the risk of a jump in z_{1t} . This term is constant because the jump size ξ_t features a time invariant distribution.

Proposition 3 characterizes the prices of consumption strips, dividend strips, and non defaultable bonds with time-to-maturity τ .

Proposition 3. *Prices of consumption strips $S_t^{con}(\tau)$, dividend strips $S_t^{div}(\tau)$, and non defaultable bonds $B_t(\tau)$ with time-to-maturity τ satisfy*

$$S_t^{con}(\tau) = \mathbb{E}_t^{\mathbb{Q}} \left(e^{-\int_t^{t+\tau} r_s ds} C_{t+\tau} \right) = e^{a_{con}(\tau) + b_{con}(\tau)^\top Y_t}, \quad (26)$$

$$S_t^{div}(\tau) = \mathbb{E}_t^{\mathbb{Q}} \left(e^{-\int_t^{t+\tau} r_s ds} D_{t+\tau} \right) = e^{a_{div}(\tau) + b_{div}(\tau)^\top Y_t}, \quad (27)$$

and

$$B_t(\tau) = \mathbb{E}_t^{\mathbb{Q}} \left(e^{-\int_t^{t+\tau} r_s ds} \right) = e^{a_B(\tau) + b_B(\tau)^\top Y_t}, \quad (28)$$

where \mathbb{Q} is the risk-neutral measure defined by Proposition 2. The deterministic functions $a_{con}(\cdot), a_{div}(\cdot), a_B(\cdot) \in \mathbb{R}$ and $b_{con}(\cdot), b_{div}(\cdot), b_B(\cdot) \in \mathbb{R}^5$ solve a system of ordinary differential equations provided in Appendix C.

Proof. See Appendix C.

The valuation ratios $S_t^{con}(\tau)/C$, $S_t^{div}(\tau)/D$, and $B_t(\tau)$ do not depend on x and, hence, are stationary. Similar to Proposition 4, an application of Itô's lemma to the dividend strip price, $S_t^{div}(\tau)$, allows us to derive the term structures of both equity return volatility, $\|\sigma_t^{div}(\tau)\|$, and risk premia $RP_t^{div}(\tau)$.

The price of equity (i.e., the claim on dividends) can be either computed by numerical integration of the dividend strip prices over the time horizon

$$P_t = \int_0^\infty S_t^{\text{div}}(\tau) d\tau, \quad (29)$$

or approximated through a log-linearization of the return process. In that case, the price satisfies the following exponential affine form: $P_t \approx D_t e^{A_d + B_d^\top Y_t}$. This expression is particularly convenient when pricing other assets such as options (Eraker and Shaliastovich, 2008).

The conditional return volatility and risk premia of a strip with time-to-maturity τ is provided in Proposition 4.

Proposition 4. *The time t return variance of a strip with time-to-maturity τ is written*

$$\|\sigma_t^{CF}(\tau)\|^2 = \|\sigma_t^{CF,c}(\tau)\|^2 + \|\sigma_t^{CF,d}(\tau)\|^2, \quad (30)$$

where $CF \in \{\text{con}, \text{div}\}$ determines either the consumption strip or the dividend strip and $\|\sigma_t^{CF,c}(\tau)\|^2$ and $\|\sigma_t^{CF,d}(\tau)\|^2$ are the return variances implied by the Brownian motion W_t and the jump process N_t , respectively. The vectors $\sigma_t^{CF,c}(\tau)$ and $\sigma_t^{CF,d}(\tau)$ satisfy

$$\sigma_t^{CF,c}(\tau) = \frac{1}{S_t^{CF}(\tau)} \left(\frac{\partial}{\partial Y} S_t^{CF}(\tau) \right)^\top \Sigma(Y_t) \quad (31)$$

and

$$\sigma_t^{CF,d}(\tau) = \sqrt{l(Y_t)^i \mathbb{E} \left[\left(e^{b_{CF}^i(\tau) J_t^i} - 1 \right)^2 \right]} = \sqrt{l(Y_t)^i (\varrho(2b_{CF}^i(\tau)) - 2\varrho(b_{CF}^i(\tau)) + 1)}. \quad (32)$$

The time t risk premia on a strip with time-to-maturity τ are written

$$RP_t^{CF}(\tau) = RP_t^{CF,c}(\tau) + RP_t^{CF,d}(\tau), \quad (33)$$

where $RP_t^{CF,c}(\tau)$ and $RP_t^{CF,d}(\tau)$ are the premia for bearing Brownian and jump risks, re-

spectively. These risk premia satisfy

$$RP_t^{CF,c}(\tau) = \sigma_t^{CF,c}(\tau)\Lambda_t^c, \quad (34)$$

and

$$RP_t^{CF,d}(\tau) = \sum_{i=1}^5 l(Y_t)^i \mathbb{E} \left[\left(e^{b_{CF}^i(\tau) J_t^i} - 1 \right) \Lambda_t^{d^i} \right] \quad (35)$$

$$= \sum_{i=1}^5 l(Y_t)^i \left(\varrho(b_{CF}^i(\tau)) - \varrho(b_{CF}^i(\tau) - \Omega^i) + \varrho(-\Omega^i) - 1 \right). \quad (36)$$

Proof. Application of Itô's lemma and Girsanov's theorem.

In Section 4, we show that the level and shape of the term structures of equity return volatility and risk premia are inherited by the level and shape of the term structure of dividend growth volatility when the EIS is large enough.

3.3. Calibration

The model is calibrated as follows. First, we fit the probability of disasters, the average size of disasters, the average size of recoveries, and the length of recoveries to the values estimated by Gourio (2008), Barro and Ursua (2008), and Nakamura et al. (2013). The empirical probability of disasters is in the range (0.028, 0.036), and the empirical size of disasters is in the range (0.22, 0.32). To be conservative, we pick the lower bounds and therefore set $\bar{\lambda} = 0.028$ and $\eta = 4.02$. The parameters $\omega = 0.108$ and $\phi_z = 0.302$ imply a 50% recovery from disasters in approximately five years. That is, 50% of the disaster size is permanent. Second, the remaining parameters are such that the yearly mean of consumption and dividend growth, the yearly volatility of consumption and dividend growth, the long-term (ten years) variance ratio of consumption and dividend growth, and the mean and volatility of the dividend share are as close as possible to their empirical counterparts. Table 2 shows the empirical and theoretical moments of consumption and dividends, and Table

3 provides the corresponding parameter values. The size and frequency of disasters imply substantial consumption growth volatility. Therefore, we set $\sigma_x = 0$ such that consumption growth is smooth and varies only with the expected growth rate, disasters, and recoveries.

[Insert Tables 2 and 3 about here.]

4. Results

In this section, we first discuss the properties of the term structure of dividend growth risk because it is the main determinant of the shape of the term structures of equity return volatility and risk premia. Then, we show how preferences and economic conditions impact the term structures of equity return volatility, equity risk premia, and interest rates. The model can simultaneously explain the shape of the term structures of equity return and interest rates, while providing a solution to the equity premium and risk-free rate puzzles. In addition, we show that the price-dividend ratio predicts future excess returns in the model, consistent with empirical findings.

4.1. Term structures of consumption and dividend growth risk

The conditional term structures of cash flow growth volatility and variance ratios are characterized in Proposition 5.

Proposition 5. *The time t cash flow growth volatility at a τ -year horizon, $vol^{CF}(t, \tau)$, is written*

$$vol^{CF}(t, \tau) = \sqrt{\frac{1}{\tau} \log \left(\frac{MGF^{CF}(t, \tau; 2)}{MGF^{CF}(t, \tau; 1)^2} \right)}, \quad (37)$$

where $CF \in \{con, div\}$ determines whether the cash flow is the consumption C or the dividend

D , and the conditional moment-generating function $MGF^{CF}(\cdot, \cdot; \cdot)$ satisfies

$$MGF^{CF}(t, \tau; u) = \mathbb{E}_t (F_{t+\tau}^u) = e^{\bar{a}_{CF}(\tau; u) + \bar{b}_{CF}(\tau; u)^\top Y_t}, \quad (38)$$

where $F = C$ if $CF = con$ and $F = D$ if $CF = div$. The deterministic functions $\bar{a}_{CF}(\cdot; \cdot) \in \mathbb{R}$ and $\bar{b}_{CF}(\cdot; \cdot) \in \mathbb{R}^5$ solve a system of ordinary differential equations provided in Appendix D. The time t cash flow growth variance ratio at a τ -year horizon, $VR^{CF}(t, \tau)$, is defined as

$$VR^{CF}(t, \tau) = \frac{vol^{CF}(t, \tau)^2}{vol^{CF}(t, 1)^2}. \quad (39)$$

Proof. See Appendix D.

Notice that $vol^{CF}(t, \tau)$ captures both the continuous and the discontinuous expected variation in the cash flow growth. Moreover, the first component of $\bar{b}_{CF}(\tau; u)$ is simply u , such that $vol^{CF}(t, \tau)$ and $VR^{CF}(t, \tau)$ do not depend on x_t .

Fig. 3 displays the term structures of dividend growth and consumption growth volatility (on the left) and the corresponding variance ratios (on the right). In line with empirical findings (see Fig. 1), the model implies strongly downward-sloping and slightly upward-sloping term structures of dividend growth volatility and consumption growth volatility, respectively. That is, accounting for recoveries helps explain the observed shape of the term structures of dividend growth and consumption growth risk.¹² The reason is that the three main components of cash flow growth risk are the disaster risk, the intensity risk, and the expected growth risk.¹³ Time variation in jump intensity, λ_t , and expected growth, μ_t , implies an

¹²Beeler and Campbell (2012), Belo, Collin-Dufresne, and Goldstein (2015), and Marfè (2014) provide empirical evidence of a decreasing term structure of dividend growth variance ratios.

¹³We do not discuss the risk implied by the Brownian motion W_{xt} because it has no effect on the slope of the term structure (i.e., this Brownian risk is proportional to the time horizon).

upward-sloping effect on the term structure because the longer the horizon is, the higher the uncertainty concerning the frequency of disasters and the propensity to grow becomes.¹⁴ Consider now the role of z_{1t} and z_{2t} . Over short horizons (e.g., one day), recoveries do not influence disaster risk. Over long horizons (e.g., 20 years), recoveries dampen the disaster risk that would have prevailed, had there been no recoveries ($\phi_z = 0$). Therefore, recovery from disasters implies a downward-sloping effect on the term structure. Overall, the downward-sloping effect of recovery dominates the upward-sloping effect of time-varying intensity and expected growth when cash flows are sufficiently exposed to disasters and recoveries, i.e., when α is sufficiently large. This generates a downward-sloping term structure of dividend growth risk and a slightly upward-sloping term structure of consumption growth risk.

[Insert Fig. 3 about here.]

In what follows, first, we analyze the model predictions concerning the term structures of equity return and interest rates. Second, we show how the model reconciles a solution to the risk-free rate and equity premium puzzles with an adequate description of the term structures of consumption growth risk, dividend growth risk, equity return, and interest rates. Third, we show that the model implies time series predictability patterns that are in line with the data.

4.2. *Term structures of equity return and interest rates*

Fig. 4 displays the term structures of return volatility (left) and risk premia (right) for the dividend and consumption strips. When the elasticity of intertemporal substitution is lower than one, the income effect dominates the substitution effect and the term structures associated with the strip on consumption are slightly upward-sloping. However, when divi-

¹⁴In the absence of recoveries, both time-varying disaster intensity and time-varying expected growth imply strongly upward-sloping term structures of cash flow growth risk and, in turn, of equity return volatility and risk premia (van Binsbergen, Brandt, and Koijen, 2012).

depends load more than consumption on disaster risk, even a moderate substitution effect (e.g. $\psi < 1$) is sufficient to generate downward-sloping term structures of dividend strips. Equity compensation is high in the short term because dividends are highly sensitive to disaster risk ($\alpha > 1$), and disaster risk concentrates at short horizons in the presence of recoveries ($\phi_z > 0$). The larger α is, the smaller the elasticity of intertemporal substitution needed to obtain the observed slopes of the term structures of equity return.

[Insert Fig. 4 about here.]

Fig. 5 describes the relation between the elasticity of intertemporal substitution and the slopes of the term structures. We observe two noteworthy results. First, equity return risk equals dividend growth risk at very short horizons ($\tau \rightarrow 0$) and inherits the negative slope of dividend growth risk. However, the larger the elasticity of intertemporal substitution, the steeper the negative slope of equity return risk becomes. Then, a moderate elasticity of intertemporal substitution (e.g., $\psi = 0.8$) helps to dampen the excess volatility of dividend growth relative to equity return at long horizons, a counterfactual implication of several asset pricing models (see Beeler and Campbell, 2012).

[Insert Fig. 5 about here.]

Second, the slope of equity risk premia is negative and the level of risk premia is decreasing with the elasticity of intertemporal substitution. Dividend strip risk premia lie between 6% and 8% when $\psi = 0.8$, and they lie between 3% and 4% when $\psi = 1.5$. As recognized by Gourio (2008), required premia for bearing disaster risk are decreasing with ψ in the presence of recovery. The reason is twofold. First, recoveries imply a decreasing term structure of dividend growth risk, meaning that risk is concentrated in the short term. Second, the investor requires a low (high) premium for bearing short-term risk when he has a preference for early (late) resolution of uncertainty. Because an increase in the elasticity of intertemporal

substitution implies a preference for earlier resolution of uncertainty, the relation between risk premia and the parameter ψ is negative. The opposite holds when the term structure of dividend growth risk is upward-sloping, i.e., when there is no recovery. In this case, risk premia increase with ψ because the larger the elasticity of intertemporal substitution is, the earlier the investor is willing to resolve uncertainty and, therefore, the larger the required compensation for bearing long-term risk.

This reasoning explains why previous disaster models (Wachter, 2013), as well as long-run risk models (Bansal and Yaron, 2004), have difficulty reconciling downward-sloping term structures with high equity risk premia. Both classes of models assume cash flow dynamics that imply an upward-sloping term structure of dividend growth risk. Therefore, these models can generate either high risk premia and upward-sloping term structures of equity return if the agent has a preference for early resolution of uncertainty ($\gamma > 1/\psi$) or low risk premia and downward-sloping term structures of equity return if the agent has a preference for late resolution of uncertainty ($\gamma < 1/\psi$).

Our setup mitigates this tension by improving the timing of consumption and dividend growth risk. Even for relatively low preference parameters ($\gamma = 5.5, \psi = 0.8$), the (partially) transitory nature of disasters ($\phi_z > 0$) and the larger sensitivity of dividends relative to consumption on disaster risk ($\alpha > 1$) are sufficient to generate high compensations and downward-sloping term structures of equity return.

As in Wachter (2013), we assume that the disaster intensity is a proxy for economic conditions. That is, good, normal, and bad times correspond to a low, moderate, and high disaster intensity. Fig. 6 shows that the term structures of equity return are downward-sloping and flatten when economic conditions improve. The fact that dividends are expected to recover in the long term implies less risk in the long term than in the short term and, therefore, downward-sloping term structures of equity return. When economic conditions are good, the disaster intensity is expected to revert back up to its long-term mean. This increases the dividend growth risk in the long term and, therefore, dampens the downward-

sloping effect implied by recoveries. As shown by van Binsbergen, Hueskes, Koijen, and Vrugt (2013), the pro-cyclical dynamics of the slope of the term structure of equity risk premia find empirical support.

[Insert Fig. 6 about here.]

Fig. 7 depicts the term structures of real interest rates implied by the model. The term structure of interest rates is downward-sloping when there are no recoveries and upward-sloping when there are recoveries. The reason is that the term structure of consumption growth risk is significantly less upward-sloping in the latter case. Because bonds are hedging instruments, bond prices are positively related to consumption growth risk, and bond yields are inversely related to it. Therefore, bond yields decrease with maturity when there are no recoveries and increase with it otherwise. This shows that recoveries not only help understand the observed shape of the term structures of equity return, but also that of the term structure of interest rates.

[Insert Fig. 7 about here.]

The term structure of real interest rates flattens as the EIS increases (bottom left of Fig. 7). The reason is that bond prices are defined as the expectation under the risk-neutral measure of a \$1 future cash flow discounted at the risk-free rate (Eq. (28)). Therefore, the term structure is flat if the risk-free rate is constant (no volatility) and steeper if the risk-free rate is volatile. Because the volatility of the risk-free rate mainly depends on the product of the volatility of the expected growth rate and the inverse of the EIS, it decreases with the EIS (Bansal and Yaron, 2004). That is, an increase in EIS implies a decrease in the volatility of the risk-free rate and, therefore, a flatter term structure of interest rates.

Consistent with the findings of Buraschi and Jiltsov (2007), the slope of the term structure of real interest rates is counter cyclical in our model (bottom right graphic of Fig. 7). In

good times, the intensity of disaster λ_t is low, which implies low consumption growth risk in the short term and therefore a small demand for short-term bonds. That is, short-term bonds are cheap and their yields high. In the longer term, investors expect the intensity to revert back up to its long-term mean, which increases long-term consumption growth risk and decreases bond yields. In bad times, the opposite holds. Consumption growth risk is high in the short term because the intensity of disaster is high. Consequently, short-term bonds are expensive and their yields low. Because investors expect the intensity of disaster to revert back down to its long-term mean, long-term consumption growth risk is low and, therefore, long-term bond yields are high.

4.3. *Equity risk premia, risk-free rate, and return predictability*

To complete the analysis, we look at the model-implied levels of the risk-free rate and equity risk premia. Table 4 shows how these two variables depend on the pairs of preference parameters γ and ψ . Risk premia increase with relative risk aversion and decrease with the elasticity of intertemporal substitution. The latter result comes from the fact that the main source of risk premia is disaster risk, which is (partially) transitory in the presence of recovery ($\phi_z > 0$). We observe that the pair $\gamma = 5.5$ and $\psi = 0.8$ leads to sizable and realistic risk premia (about 6.3%) and a low risk-free rate (about 0.9%). Therefore, the model reconciles the risk-free rate and equity premium puzzle with the recent empirical evidence concerning the term structures of equity (van Binsbergen, Brandt, and Koijen, 2012).¹⁵ The levered exposition of dividends to disaster risk is a property of the data (see Section 2), which helps obtain downward-sloping term structures of equity return for values of EIS smaller than one. Because risk premia are decreasing with the EIS in the presence of recovery, dividend leverage ($\alpha > 1$) helps match, both qualitatively and quantitatively, the observed term structures of

¹⁵These parameters imply a relatively low equity volatility (about 6%). The level of volatility can be increased by decreasing the EIS, at the expense of generating a too low risk-free rate and too high risk premia.

equity return and the average level of risk premia.

[Insert Table 4 about here.]

A debate is ongoing about the values of the EIS used in theoretical models and those estimated using real data (see Hall (1988) and Epstein, Farhi, and Strzalecki (2014)). Theoretical models are usually very sensitive to the EIS and generate realistic price patterns only under specific restrictions on the EIS (e.g., Bansal and Yaron, 2004). In what follows, we want to verify whether our results are robust to the choice of preference parameters and, in particular, to the choice of the EIS. Fig. 8 shows the pairs of relative risk aversion and time discount factor (γ, δ) , which allow us to match the empirical levels of the risk-free rate and equity risk premia for values of the EIS in the range $(1/2, 2)$. We require a risk-free rate of 0.6% and risk premia of 6.5% (Constantinides and Ghosh, 2011).

[Insert Fig. 8 about here.]

Several results are noteworthy. First, the model can quantitatively solve the risk-free rate and equity premium puzzle for any value of EIS within this range. Second, the pairs (γ, δ) required by the model belong to the ranges $\delta \in (0.95, 0.98)$ and $\gamma \in (4.4, 6.6)$, which are considered to be plausible in the existing literature. Because risk premia are decreasing with the EIS when there are recoveries, an increase in both the EIS and relative risk aversion is needed to match observed risk premia. Third, for each value of the EIS within this range, the pair (γ, δ) required to match the observed risk-free rate and risk premia also implies downward-sloping term structures of equity return. That is, the speed of recovery from disasters implies a shift of equity risk toward the short term that is robust to a wide range of preference parameters. This provides further support to the idea that recovery from disasters in macroeconomic fundamentals is a key driver of asset prices.

In what follows, we investigate the predictability of excess stock returns implied by our

model. To do so, we regress long-horizon excess returns on the log price-dividend ratio. The regression is performed on returns measured over horizons ranging from one to seven years. In each of our one thousand simulations, the price-dividend ratio and excess returns are simulated at the monthly frequency over a 60-year horizon. The choice of the frequency and horizon matches that of most of the empirical literature on predictive regressions.

Panel A of Table 5 shows that the model reproduces the negative and statistically significant intertemporal relation between the current valuation ratios and future excess returns that we observe in actual data (Cochrane, 2008; Lettau and van Nieuwerburgh, 2008; van Binsbergen and Koijen, 2010). In contrast with empirical evidence, the forecasting power of the price-dividend ratio decreases with the horizon because most of the predictability comes out from transitory risk in fundamentals.¹⁶ Therefore, recoveries from disasters help understand the timing of dividend growth risk and the shape of the term structures of equity return and interest rates, but they are likely not sufficient to accurately explain the observed positive relation between the explanatory power of the price-dividend ratio and the horizon.

[Insert Table 5 about here.]

Because disasters are rare events, it is noteworthy to understand whether the model leads to stock-return predictability over time periods in which disasters do not realize. Panel B of Table 5 shows the regression results when disasters are turned off from the simulations. A strong negative intertemporal relation between prices and future excess returns is still observable. This obtains because investors take into account the possibility that a disaster followed by a recovery could occur. Moreover, the probability of such an event is mean reverting because the disaster intensity follows a square-root process.

¹⁶Lettau and Wachter (2007) suggest some caution when discussing the empirical properties of R^2 statistics because they are unstable over different sample periods.

5. Slow disasters and the term structures of equity

Nakamura, Steinsson, Barro, and Ursua (2013) show that rare disasters do not affect consumption instantaneously but gradually. Their estimation shows that disasters take approximately four years to fully realize.¹⁷ The aim of this section is twofold. First, we model “slow disasters” in a parsimonious way that guarantees analytical tractability. Second, we show that the main message of the paper –disaster recovery leads to downward-sloping term structures of equity return– is robust to the property of “slow disasters”.

We model the joint dynamics of consumption and dividends as

$$\log C_t = x_t + s_{1t} \tag{40}$$

and

$$\log D_t = x_t + s_{2t} + \log d_0, \quad d_0 \in (0, 1), \tag{41}$$

with

$$dx_t = (\mu - \frac{1}{2}\sigma_x^2 + \omega z_{1t})dt + \sigma_x dW_{xt}, \tag{42}$$

$$ds_{it} = \phi_s(z_{it} - s_{it})dt + \sigma_{si}dW_{st}, \tag{43}$$

and

$$dz_{it} = -\phi_z z_{it}dt + \alpha_i \xi_t dN_{zt}, \tag{44}$$

where $i \in \{1, 2\}$, $\alpha_1 = 0$, and $\alpha_2 = \alpha$. Consumption and dividends are co-integrated because

¹⁷Four years is the median estimated time for disasters to fully realize, and six years is the average estimated time. We choose four years instead of six years because the model fits the moments of consumption, dividends, and dividend share better in this case.

s_{1t} and s_{2t} are stationary, dividends are more sensitive than consumption to disasters and recoveries when $\alpha > 1$, disasters partially recover when $\phi_z > 0$ and $\omega > 0$, and the speed of reversion ϕ_s governs the length of the disaster period. For simplicity, we turn off time variation in expected growth μ and disaster intensity λ . Because the above dynamics belong to the affine class, we can again follow the methodology of Eraker and Shaliastovich (2008) and solve for prices as in Section 3.

In this model, the parameters λ , η , ω , ϕ_z , and ϕ_s characterize disasters and recoveries. We choose these parameters to match the probability of a disaster occurring, the size of a disaster, the length of the disaster period, the size of the recovery, and the length of the recovery period. The remaining parameters $(\mu, \sigma_x, \sigma_{s1}, \sigma_{s2}, \alpha, d_0)$ are set to match the yearly mean and volatility of consumption and dividend growth, their ten year variance ratios, and the mean and volatility of the dividend share of consumption. Empirical moments, theoretical moments, and parameter values are provided in Appendix F. The model fits the data well, which shows that accounting for “slow disasters” does not diminish the model’s ability to match the timing of fundamentals risk.

The upper left graphic of Fig. 9 depicts the deterministic evolution of log-consumption conditional on a disaster occurring at time 0 ($dN_{z0} = 1$). It shows that our parsimonious dynamics are flexible enough to reproduce the typical path of consumption documented by Nakamura, Steinsson, Barro, and Ursua (2013).

[Insert Fig. 9 about here.]

The upper right graphic of Fig. 9 reports the term structures of consumption and dividend growth risk. Consistent with empirical findings, dividend growth risk decreases with the horizon and consumption growth risk is almost constant.

We conclude our analysis by choosing preference parameters that guarantee downward sloping term structures of equity return volatility and risk premia and reasonable values for the risk-free rate and equity premium. On the one hand, disaster recovery ($\phi_z > 0$) and

levered exposition of dividends to disasters ($\alpha > 1$) imply that prices inherit the negative slope of the term structure of dividend growth risk for a large range of preference parameters. On the other hand, “slow disasters” imply that a high elasticity of intertemporal substitution is needed to generate sizable risk premia. Slow disasters imply that consumption growth is safer than in our main model at short horizons. Therefore, similar risk premia can be obtained only if the investor has a preference for early resolution of uncertainty. As in Nakamura, Steinsson, Barro, and Ursua (2013), we choose $\psi = 2$ and then set $\delta = 0.965$ and $\gamma = 8.5$. The lower left and right graphics of Fig. 9 show the term structures of equity return volatility and risk premia, respectively. The term structure of equity return volatility is downward-sloping for all horizons, and the term structure of risk premia is downward-sloping as long as the horizon is larger than about three years. The term structure of risk premia is upward-sloping in the very short term because disasters are not instantaneous in the present model.

The above choice of preference parameters leads to risk premia of about 4.24% and a risk free rate of about 1.52%. Although these numbers do not perfectly match their empirical counterparts, they suggest that disaster risk as a key driver of equity risk premia can be reconciled with the empirical evidence about the term structures of both fundamentals and equity return. This model, however, implies negative bond yields for most horizons, a downward-sloping term structure of interest rates for horizons ranging from zero to five years, and an upward-sloping term structure of interest rate for horizons larger than five years. To summarize, the observed negative slopes of term structures of equity return, the low risk-free rate, and high risk premia are robust to the property of “slow disasters”, but “slow disasters” imply a counterfactual term structure of interest rates.

6. Conclusion

Empirical research provides evidence that the term structures of equity return volatility and risk premia are downward-sloping. This finding is particularly important provided that leading asset pricing models (Bansal and Yaron, 2004; Gabaix, 2012; Wachter, 2013) imply strongly upward-sloping term structures of equity return. In this paper, we show that the term structures of equity return obtained in a rare disasters model are downward-sloping when accounting for post-disaster recoveries, a feature of the data disregarded by most of the literature. In the presence of recoveries, disaster risk concentrates in the short term, which implies downward-sloping term structures of dividend growth risk, equity return volatility, and equity risk premia.

The model accounts for the observed slightly upward-sloping term structure of consumption growth risk (due to time-varying expected growth and disaster intensity) and the downward-sloping term structure of dividend growth risk (due to the levered exposition of dividends to disasters and recoveries). It reconciles high equity risk premia and a low risk-free rate with the shape of the term structures of equity return and interest rates for a broad range of realistic preference parameters.

Appendix A. Proof of Proposition 1

Following Eraker and Shaliastovich (2008), the state-price density satisfies

$$d \log M_t = (\theta \log \delta - (\theta - 1) \log k_1 + (\theta - 1)(k_1 - 1)B'_c(Y_t - \mu_Y)dt - \Omega' dY_t, \quad (45)$$

where $\mu_Y = (0, 0, 0, \bar{\lambda}, \bar{\mu})^\top$ and $\Omega = \gamma(1, 1, 0, 0, 0)^\top + (1 - \theta)k_1 B_c$.

The coefficients $A_c \in \mathbb{R}$, $B_c \in \mathbb{R}^5$, and $k_1 \in \mathbb{R}$ characterizing the wealth, V_t , and the state-price density, M_t , solve the following system of equations:

$$0 = \mathcal{K}^\top \chi - \theta(1 - k_1)B_c + \frac{1}{2}\chi^\top H \chi + l_1^\top (\varrho(\chi) - 1), \quad (46)$$

$$0 = \theta(\log \delta + k_0 - (1 - k_1)A_c) + \mathcal{M}^\top \chi + \frac{1}{2}\chi^\top h \chi + l_0^\top (\varrho(\chi) - 1), \quad (47)$$

and

$$\theta \log k_1 = \theta(\log \delta + (1 - k_1)B_c^\top \mu_Y) + \mathcal{M}^\top \chi + \frac{1}{2}\chi^\top h \chi + l_0^\top (\varrho(\chi) - 1), \quad (48)$$

where $\chi = \theta \left((1 - \frac{1}{\psi})(1, 1, 0, 0, 0)^\top + k_1 B_c \right)$, $k_0 = -\log((1 - k_1)^{1-k_1} k_1^{k_1})$.

□

Appendix B. Proof of Proposition 2

Following Eraker and Shaliastovich (2008), the coefficients $\Phi_0 \in \mathbb{R}$ and $\Phi_1 \in \mathbb{R}^5$ characterizing the risk-free rate r_t solve the following system of equations:

$$\Phi_1 = (1 - \theta)(k_1 - 1)B_c + \mathcal{K}^\top \Omega - \frac{1}{2}\Omega^\top H \Omega - l_1^\top (\varrho(-\Omega) - 1) \quad (49)$$

and

$$\Phi_0 = -\theta \log \delta + (\theta - 1)(\log k_1 + (k_1 - 1)B_c^\top \mu_Y) + \mathcal{M}^\top \Omega - \frac{1}{2}\Omega^\top h\Omega - l_0^\top (\varrho(-\Omega) - 1). \quad (50)$$

□

Appendix C. Proof of Proposition 3

Following Eraker and Shaliastovich (2008), the deterministic functions $a_j(\cdot) \in \mathbb{R}$ and $b_j(\cdot) \in \mathbb{R}^5$, $j \in \{\text{con}, \text{div}, B\}$ characterizing the price of strips and non-defaultable bonds solve the following system of ordinary differential equations:

$$b'_j(\tau) = -\Phi_1 + \mathcal{K}^{\mathbb{Q}\top} b_j(\tau) + \frac{1}{2}b_j(\tau)^\top H b_j(\tau) + l_1^{\mathbb{Q}\top} (\varrho^{\mathbb{Q}}(b_j(\tau)) - 1) \quad (51)$$

and

$$a'_j(\tau) = -\Phi_0 + \mathcal{M}^{\mathbb{Q}\top} b_j(\tau) + \frac{1}{2}b_j(\tau)^\top h b_j(\tau) + l_0^{\mathbb{Q}\top} (\varrho^{\mathbb{Q}}(b_j(\tau)) - 1), \quad (52)$$

subject to $a_{\text{con}}(0) = 0$, $b_{\text{con}}(0) = (1, 1, 0, 0, 0)^\top$, $a_{\text{div}}(0) = 0$, $b_{\text{div}}(0) = (1, 0, 1, 0, 0)^\top$, $a_B(0) = 0$, and $b_B(0) = (0, 0, 0, 0, 0)^\top$. The coefficients $\mathcal{M}^{\mathbb{Q}} \in \mathbb{R}^5$, $\mathcal{K}^{\mathbb{Q}} \in \mathbb{R}^{5 \times 5}$, $l_0^{\mathbb{Q}} \in \mathbb{R}^5$, $l_1^{\mathbb{Q}} \in \mathbb{R}^{5 \times 5}$, and $\varrho^{\mathbb{Q}}(\cdot) \in \mathbb{R}^5$ satisfy

$$\mathcal{M}^{\mathbb{Q}} = \mathcal{M} - h\Omega, \quad (53)$$

$$\mathcal{K}^{\mathbb{Q}} = \mathcal{K} - H\Omega, \quad (54)$$

$$l^{\mathbb{Q}}(Y_t) = l(Y_t) \cdot \varrho(-\Omega) \equiv l_0^{\mathbb{Q}} + l_1^{\mathbb{Q}} Y_t, \quad (55)$$

and

$$\varrho^{\mathbb{Q}}(u) = \varrho(u - \Omega) / \varrho(-\Omega). \quad (56)$$

□

Appendix D. Proof of Proposition 5

Following Duffie, Pan, and Singleton (2000), the deterministic functions $\bar{a}_{CF}(\cdot; \cdot) \in \mathbb{R}$ and $\bar{b}_{CF}(\cdot; \cdot) \in \mathbb{R}^5$ characterizing the moment-generation function $\text{MGF}(\cdot, \cdot; \cdot)$ solve the following system of ordinary differential equations:

$$\bar{b}'_{CF}(\tau; u) = \mathcal{K}^\top \bar{b}_{CF}(\tau; u) + \frac{1}{2} \bar{b}_{CF}(\tau; u)^\top H \bar{b}_{CF}(\tau; u) + l_1^\top (\varrho(\bar{b}_{CF}(\tau; u)) - 1) \quad (57)$$

and

$$\bar{a}'_{CF}(\tau; u) = \mathcal{M}^\top \bar{b}_{CF}(\tau; u) + \frac{1}{2} \bar{b}_{CF}(\tau; u)^\top h \bar{b}_{CF}(\tau; u) + l_0^\top (\varrho(\bar{b}_{CF}(\tau; u)) - 1), \quad (58)$$

subject to $\bar{a}_{CF}(0) = 0$, $\bar{b}_{CF}(0) = (u, u, 0, 0, 0)^\top$ if $CF = \text{con}$, and $\bar{b}_{CF}(0) = (u, 0, u, 0, 0)^\top$ if $CF = \text{div}$.

□

Appendix E. Accuracy of the approximation

In this section, we investigate whether disasters (i.e., jumps in consumption) affect the accuracy of the log-linear approximation of the wealth return dynamics. To do so, we note that the sum of discounted (with the state-price density M_t) wealth and cumulative discounted past consumption is a martingale, G_t , under the objective probability measure.

That is,

$$G_t \equiv M_t V_t + \int_0^t M_s C_s ds \quad (59)$$

satisfies

$$0 = \frac{1}{dt} \mathbb{E}_t [dG_t] = \frac{1}{dt} \mathbb{E}_t [d(M_t V_t)] + M_t C_t. \quad (60)$$

Substituting

$$V_t = C_t e^{v_c(z_{1t}, \lambda_t, \mu_t)} \quad (61)$$

in Eq. (60) yields a partial differential equation (PDE) for the log wealth-consumption ratio $v_c(z_{1t}, \lambda_t, \mu_t)$. Proposition 1 states that the log wealth-consumption ratio is approximated as¹⁸

$$v_c(z_{1t}, \lambda_t, \mu_t) \approx A_c + B_c^\top Y = A_c + B_{c,2} z_{1t} + B_{c,4} \lambda_t + B_{c,5} \mu_t. \quad (62)$$

Therefore, substituting Eq. (62) in the right-hand side of Eq. (60) yields PDE residuals that are different from zero unless the approximation is in fact the true solution of the PDE.

Fig. E1 depicts the PDE residuals (as a fraction of $M_t V_t$) over the state space. The solid line is obtained by setting the parameters of the model to their baseline values provided in Table 3. The dashed line differs from the solid line in one respect, namely, the size of the disaster is set to zero ($\eta \rightarrow \infty$). The residuals obtained with our baseline calibration and without disasters are of the same order of magnitude, showing that the log-linear approximation remains accurate in the presence of disasters.¹⁹

¹⁸The loadings $B_{c,1}$ and $B_{c,3}$ on x_t and z_{2t} are equal to zero.

¹⁹Campbell, Lo, and MacKinlay (1997) and Bansal, Kiku, and Yaron (2012) show that the log-linear approximation is highly accurate when shocks are normally distributed.

[Insert Fig. E1 about here.]

Appendix F. Calibration of the slow disasters model

Table F1 reports the parameter values of the slow disasters model described in Section 5. The parameters are calibrated to match the empirical moments of consumption and dividends provided in the second column of Table F2. The corresponding theoretical moments are provided in the third column of Table F2.

[Insert Tables F1 and F2 about here.]

References

- Ai, H., Croce, M. M., Diercks, A. M., Li, K., 2012. Production-based term structure of equity returns. Working paper, University of North Carolina.
- Bansal, R., Kiku, D., Yaron, A., 2012. An empirical evaluation of the long-run risks model for asset prices. *Critical Finance Review* 1, 183–221.
- Bansal, R., Yaron, A., 2004. Risks for the long run: a potential resolution of asset pricing puzzles. *Journal of Finance* 59, 1481–1509.
- Barro, R., Ursua, J., 2008. Macroeconomic crises since 1870. *Brookings Papers on Economic Activity* Spring, 255–350.
- Beeler, J., Campbell, J. Y., 2012. The long-run risks model and aggregate asset prices: an empirical assessment. *Critical Finance Review* 1, 141–182.
- Belo, F., Collin-Dufresne, P., Goldstein, R. S., 2015. Dividend dynamics and the term structure of dividend strips. *Journal of Finance* 70, 1115–1160.
- Berkman, H., Jacobsen, B., Lee, J. B., 2011. Time-varying rare disaster risk and stock returns. *Journal of Financial Economics* 101, 313–332.
- Berrada, T., Detemple, J., Rindisbacher, M., 2013. Asset pricing with regime-dependent preferences and learning. Working paper, Swiss Finance Institute.
- Buraschi, A., Jiltsov, A., 2007. Habit formation and macroeconomic models of the term structure of interest rates. *Journal of Finance* 62, 3009–3063.
- Campbell, J., Lo, A., MacKinlay, C., 1997. *The Econometrics of Financial Markets*. Princeton University Press, Princeton, NJ.
- Campbell, J. Y., Shiller, R. J., 1988. Stock prices, earnings, and expected dividends. *Journal of Finance* 43, 661–676.

- Cavallo, E., Galiani, S., Noy, I., Pantano, J., 2013. Catastrophic natural disasters and economic growth. *Review of Economics and Statistics* 95, 1549–1561.
- Cochrane, J. H., 2008. The dog that did not bark: a defense of return predictability. *Review of Financial Studies* 21, 1533–1575.
- Constantinides, G. M., Ghosh, A., 2011. Asset pricing tests with long-run risks in consumption growth. *Review of Asset Pricing Studies* 1, 96–136.
- Croce, M. M., Lettau, M., Ludvigson, S. C., 2015. Investor information, long-run risk, and the term structure of equity. *Review of Financial Studies* 28, 706–742.
- Duffie, D., Pan, J., Singleton, K., 2000. Transform analysis and asset pricing for affine jump diffusions. *Econometrica* 68, 1343–1376.
- Epstein, L. G., Farhi, E., Strzalecki, T., 2014. How much would you pay to resolve long-run risk? *American Economic Review* 104, 2680–2697.
- Epstein, L. G., Zin, S. E., 1989. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical framework. *Econometrica* 57, 937–969.
- Eraker, B., Shaliastovich, I., 2008. An equilibrium guide to designing affine pricing models. *Mathematical Finance* 18, 519–543.
- Gabaix, X., 2012. Variable rare disasters: an exactly solved framework for ten puzzles in macro-finance. *Quarterly Journal of Economics* 127, 645–700.
- Gourio, F., 2008. Disasters and recoveries. *American Economic Review* 98, 68–73.
- Gourio, F., 2012. Disaster risk and business cycles. *American Economic Review* 102, 2734–2766.
- Hall, R. E., 1988. Intertemporal substitution in consumption. *Journal of Political Economy* 96, 339–357.

- Lettau, M., Ludvigson, S. C., 2005. Expected returns and expected dividend growth. *Journal of Financial Economics* 76, 583–626.
- Lettau, M., Ludvigson, S. C., 2014. Shocks and crashes. *NBER Macroeconomics Annual* 28, 293–354.
- Lettau, M., van Nieuwerburgh, S., 2008. Reconciling the return predictability evidence. *Review of Financial Studies* 21, 1607–1652.
- Lettau, M., Wachter, J. A., 2007. Why is long-horizon equity less risky? a duration-based explanation of the value premium. *Journal of Finance* 62, 55–92.
- Lettau, M., Wachter, J. A., 2011. The term structures of equity and interest rates. *Journal of Financial Economics* 101, 90–113.
- Longstaff, F. A., Piazzesi, M., 2004. Corporate earnings and the equity premium. *Journal of Financial Economics* 74, 401–421.
- Lucas, R. E., 1978. Asset prices in an exchange economy. *Econometrica* 46, 1429–1445.
- Marfè, R., 2014. Income insurance and the equilibrium term structure of equity. Working paper, Collegio Carlo Alberto.
- Marfè, R., 2016. Corporate fraction and the equilibrium term structure of equity risk. *Review of Finance* 20, 855–905.
- Martin, I. W. R., 2013. Consumption-based asset pricing with higher cumulants. *Review of Economic Studies* 80, 745–773.
- Nakamura, E., Steinsson, J., Barro, R., Ursua, J., 2013. Crises and recoveries in an empirical model of consumption disasters. *American Economic Journal: Macroeconomics* 5, 35–74.
- Tsai, J., Wachter, J. A., 2015. Disaster risk and its implications for asset pricing. *Annual Review of Financial Economics* 7, 219–252.

- van Binsbergen, J., Brandt, M., Koijen, R., 2012. On the timing and pricing of dividends. *American Economic Review* 102, 1596–1618.
- van Binsbergen, J., Hueskes, W., Koijen, R., Vrugt, E., 2013. Equity yields. *Journal of Financial Economics* 110, 503–519.
- van Binsbergen, J. H., Koijen, R., 2016. The term structure of returns: facts and theory. *Journal of Financial Economics*, forthcoming .
- van Binsbergen, J. H., Koijen, R. S. J., 2010. Predictive regressions: a present-value approach. *Journal of Finance* 65, 1439–1471.
- Wachter, J. A., 2013. Can time-varying risk of rare disasters explain aggregate stock market volatility? *Journal of Finance* 68, 987–1035.

Table 1

Properties of US consumption and dividends.

Panel A reports the mean, standard deviation, first order autocorrelation, and cross-correlations of real consumption growth rates (Δc), real dividend growth rates (Δd), and dividend share of consumption (D/C). Panel B reports the Johansen test (one lag, constant trend) for the number of co-integrating equations in a vector error correction model (VECM) of consumption and dividends. Panel C reports the estimates and Newey West t-statistics obtained by regressing the change in the dividend share ($\Delta D/C$) on its lagged value. Panel D reports the estimates and Newey-West t-statistics obtained by regressing the change in the dividend share ($\Delta D/C$) on the disaster dummy (Dis) and the recovery dummy (Rec). A disaster period is defined as years in which growth is smaller than two standard deviation below the average as well as the subsequent negative growth years. A recovery period is defined as the observation following a disaster period. Statistical significance at the 10%, 5%, and 1% level is *, **, and ***, respectively. Data are on the sample 1929–2013 at yearly frequency from the national income and product accounts (NIPA) tables 1.1.6, 1.1.10, and 1.11. SBIC = Schwarz Bayesian information criterion; HQIC = Hannan and Quinn information criterion.

Panel A Summary statistics

Variable	Mean	Standard deviation	AC(1)
Consumption growth Δc	0.0325	0.0248	0.4956
Dividend growth Δd	0.0290	0.1501	-0.1519
Dividend share D/C	0.0477	0.0136	0.7748
			Correlation
Consumption growth and dividend growth			0.4182
Consumption growth and lagged dividend share			-0.3867
Dividend growth and lagged dividend share			-0.3841

Panel B Co-integration

Johansen test (one lag, constant trend)					
Rank	Parameters	Log likelihood	Eigenvalue	SBIC	HQIC
Rank = 0	2	-739.621		22.886	22.845
Rank = 1	5	-719.837	0.456	22.469*	22.368*

Panel C Dividend share stationarity

$$\Delta D/C_t = b_0 + b_1 D/C_{t-1} + \epsilon_t$$

Estimates	b_0 (t-statistic)	b_1 (t-statistic)	Adjusted R ²
(1)	0.010*** (3.252)	-0.215*** (-3.736)	0.117

Panel D Dividend share, disasters, and recoveries

$$\Delta D/C_t = b_0 + b_1 \text{Dis}_t + b_2 \text{Rec}_t + \epsilon_t$$

Estimates	Dis (t-statistic)	Rec (t-statistic)	Adjusted R ²
(1)	-0.018*** (-4.729)		0.327
(2)		0.016** (2.281)	0.159
(3)	-0.017*** (-4.743)	0.014** (2.131)	0.463

Table 2

Empirical and theoretical moments of consumption and dividends.
 Consumption and dividend data are on the sample 1929–2013 at yearly frequency from the national income and product accounts (NIPA) tables 1.1.6 and 1.10.

Moment	Empirical value	Theoretical value
Consumption disasters		
Probability of occurrence	0.028	0.028
Size of the drop	0.22	0.22
Size of recovery	0.5	0.5
Length of recovery (years)	~ 5	~ 5
Consumption growth		
Mean	0.03	0.03
Volatility	0.025	0.04
Ten-year variance ratio	1.01	1.06
Dividend growth		
Mean	0.03	0.03
Volatility	0.15	0.15
Ten-year variance ratio	0.37	0.29
Dividend share		
Mean	0.048	0.05
Volatility	0.014	0.011

Table 3
Calibration

Parameter	Symbol	Value
Permanent shock		
Volatility of consumption growth	σ_x	0
Long-run expected growth	$\bar{\mu}$	0.033
Volatility of expected growth	σ_μ	0.01
Speed of reversion of expected growth	ϕ_μ	0.2
Disasters		
Speed of reversion of jump intensity	ϕ_λ	0.2
Long-term jump intensity	$\bar{\lambda}$	0.028
Volatility of jump intensity	σ_λ	0.1
Jump size parameter	η	4.02
Permanent disaster parameter	ω	0.108
Speed of recovery	ϕ_z	0.302
Dividends		
Volatility of dividend growth	σ_z	0.16
Leverage parameter	α	2
Steady state dividend share	d_0	0.05
Preferences		
Relative risk aversion	γ	5.5
Elasticity of intertemporal substitution (EIS)	ψ	0.8
Time discount factor	δ	0.963

Table 4

Risk-free rate and risk premia versus preference parameters: comparative statics.
 Unless stated otherwise, state variables are $x_t = z_{1t} = z_{2t} = 0$, $\lambda_t = \bar{\lambda}$, and $\mu_t = \bar{\mu}$ and the calibration is provided in Table 3.

Relative risk aversion	$\psi = 0.8$		$\psi = 1$		$\psi = 1.5$	
	Risk-free rate	Risk premium	Risk-free rate	Risk premium	Risk-free rate	Risk premium
$\gamma = 4$	4.79%	2.42%	4.68%	1.91%	4.34%	1.47%
$\gamma = 5.5$	0.87%	6.30%	2.09%	4.54%	2.82%	3.11%
$\gamma = 7$	-28.24%	34.35%	-10.73%	16.89%	-2.72%	8.60%

Table 5

Long-horizon predictability of excess stock returns.

The table reports the coefficient, t-statistics and adjusted R^2 of the regressions of cumulative excess returns over the horizons of one, two, three, five and seven years on the current log price-dividend ratio. Panel A reports the estimates of the full model, and Panel B reports the estimates when disasters are turned off from the simulations. One thousand simulations are run at the monthly frequency over a 60-year horizon. Initial state variables are $x_t = z_{1t} = z_{2t} = 0$, $\lambda_t = \bar{\lambda}$, and $\mu_t = \bar{\mu}$, and the calibration is provided in Table 3.

<i>Panel A Model simulations with realized disasters</i>					
	Horizon (years)				
Log price-dividend ratio	1	2	3	5	7
Coefficient	-0.287	-0.479	-0.604	-0.715	-0.728
t-stat	-16.1	-14.5	-12.6	-9.3	-6.9
adjusted R^2	0.27	0.24	0.20	0.14	0.11

<i>Panel B Model simulations without realized disasters</i>					
	Horizon (years)				
Log price-dividend ratio	1	2	3	5	7
Coefficient	-0.310	-0.519	-0.657	-0.789	-0.794
t-stat	-19.4	-16.7	-14.2	-10.5	-7.8
adjusted R^2	0.34	0.28	0.24	0.17	0.12

Table F1
Calibration of the slow disasters model.

Parameter	Symbol	Value
Permanent shock		
Volatility of consumption growth	σ_x	0
Long-run expected growth	μ	0.032
Disasters		
Long-term jump intensity	λ	0.028
Jump size parameter	η	2
Permanent jump size parameter	ω	0.05
Speed of recovery	ϕ_z	0.33
Speed of disaster realization	ϕ_s	0.33
Transitory noise in consumption growth	σ_{s1}	0.037
Transitory noise in dividend growth	σ_{s2}	0.17
Dividends		
Leverage parameter	α	2
Steady state dividend share	d_0	0.05
Preferences		
Relative risk aversion	γ	8.5
Elasticity of intertemporal substitution (EIS)	ψ	2
Time discount factor	δ	0.965

Table F2

Empirical and theoretical moments of consumption and dividends.

Empirical moments are computed using consumption and dividend data on the sample 1929–2013 at yearly frequency from the national income and product accounts (NIPA) tables 1.1.6, 1.1.10, and 1.11. Theoretical moments are computed using the slow disasters model described in Section 5.

Moment	Empirical value	Theoretical value
Consumption disasters		
Probability of occurrence	0.028	0.028
Size of the drop	0.22	0.23
Size of recovery	0.5	0.55
Length of recovery (years)	~ 5	~ 7
Length of disaster (years)	~ 4	~ 4
Consumption growth		
Mean	0.03	0.03
Volatility	0.025	0.036
Ten-year variance ratio	1.01	1.04
Dividend growth		
Mean	0.03	0.03
Volatility	0.15	0.148
Ten-year variance ratio	0.37	0.31
Dividend share		
Mean	0.048	0.049
Volatility	0.014	0.009

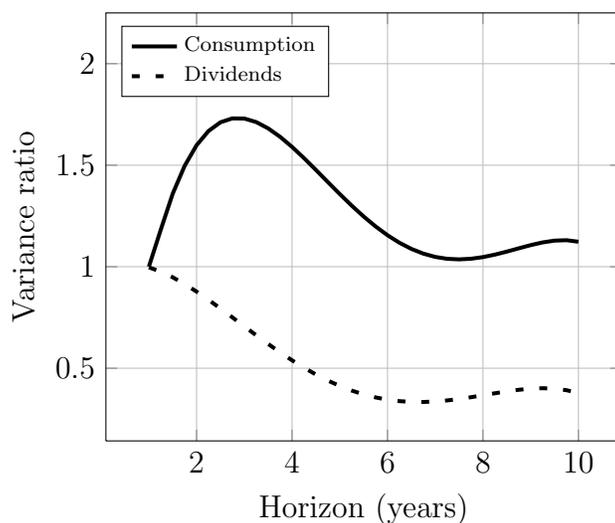


Fig. 1 Variance ratios of US consumption and dividend growth.

The figure plots the variance ratios of real US aggregate consumption (non-durable goods and services) growth and real US aggregate dividend growth. The computation of variance ratios accounts for heteroskedasticity and overlapping observations (Campbell, Lo, and MacKinlay, 1997, pp. 48–55). Data are on the sample 1929–2013 at yearly frequency from the national income and product accounts (NIPA) tables 1.1.6, 1.1.10, and 1.11.

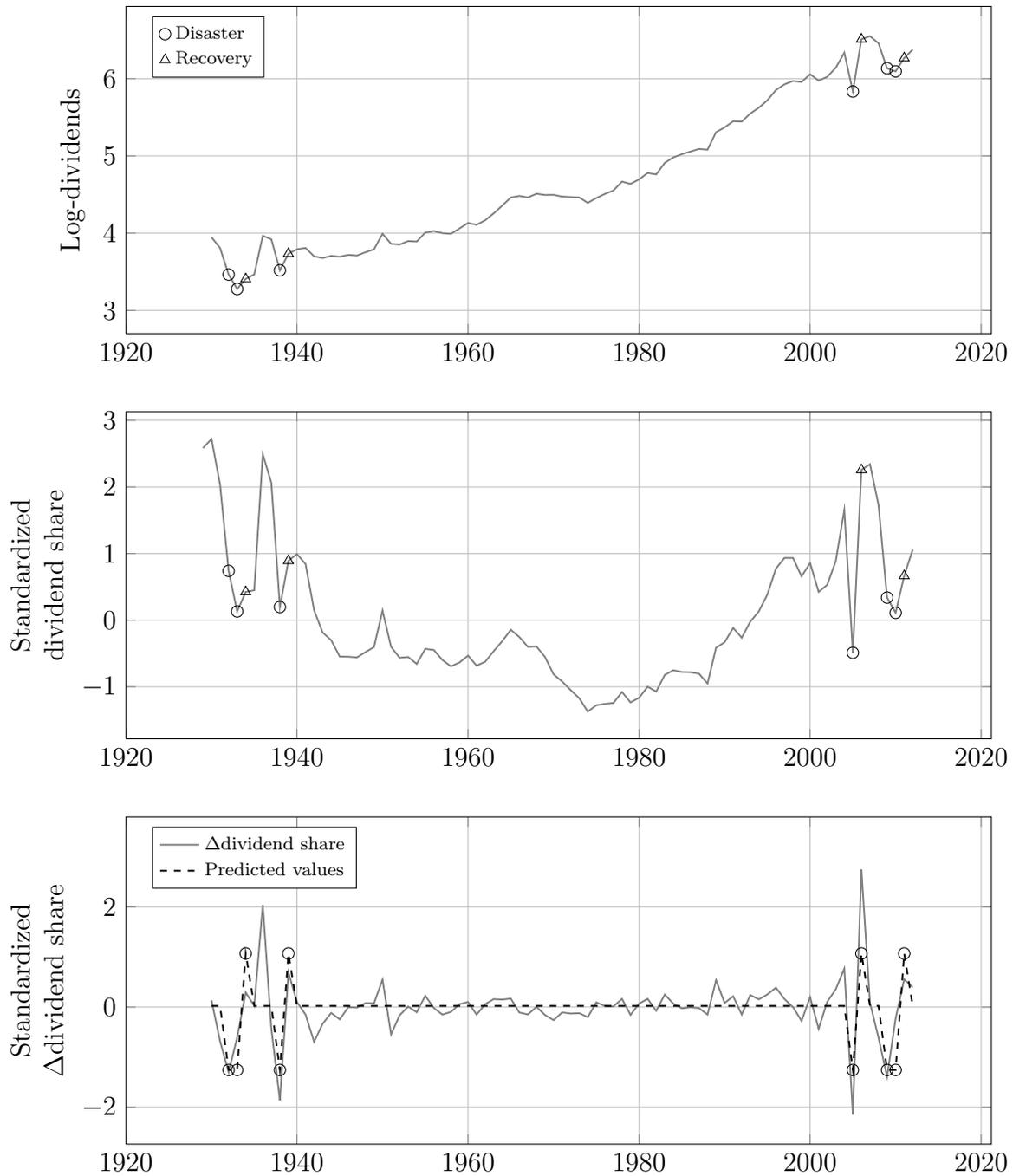


Fig. 2 US dividends and dividend share during disasters and recoveries.

The upper and middle graphics display the time series of the logarithm of US aggregate dividends and the standardized US dividend share of consumption. Circles denote disasters, i.e. years in which the dividend growth rate is smaller than two standard deviations below the average (and subsequent negative growth years). Triangles denote observations one year after disasters. The lower graphic displays the time series of the standardized changes in the dividend share and the predicted values obtained from regressing the change in the dividend share on the disaster dummy and the recovery dummy (see Panel D of Table 1). Data are on the sample 1929–2013 at yearly frequency from the national income and product accounts (NIPA) tables 1.1.6, 1.1.10 and 1.11.

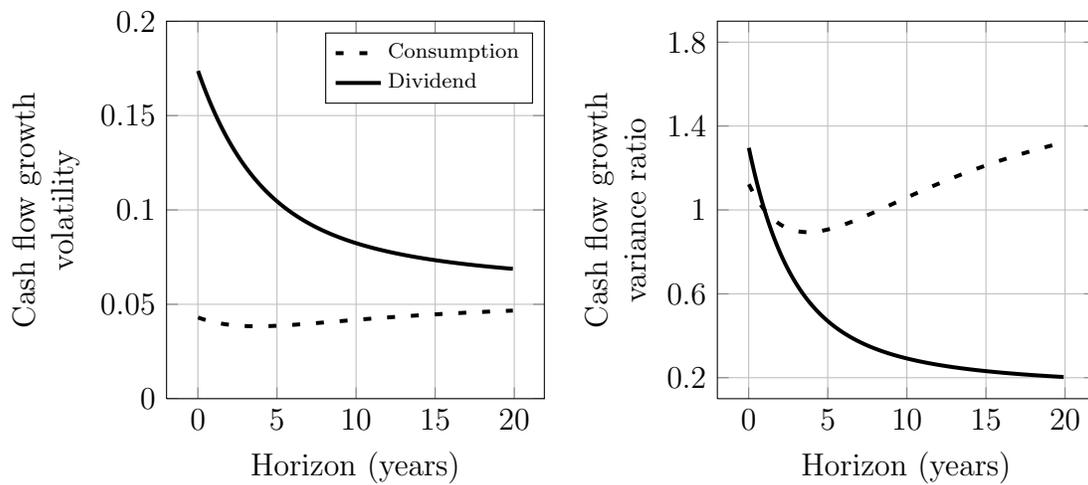


Fig. 3 Term structure of volatility and variance ratio for consumption and dividend growth.

The left graphic depicts the volatility of consumption and dividend growth for horizons ranging from 0 to 20 years. The right graphic depicts the corresponding variance ratios. That is, the τ -year variance relative to the one year variance. State variables are $x_t = z_{1t} = z_{2t} = 0$, $\lambda_t = \bar{\lambda}$, and $\mu_t = \bar{\mu}$, and the calibration is provided in Table 3.

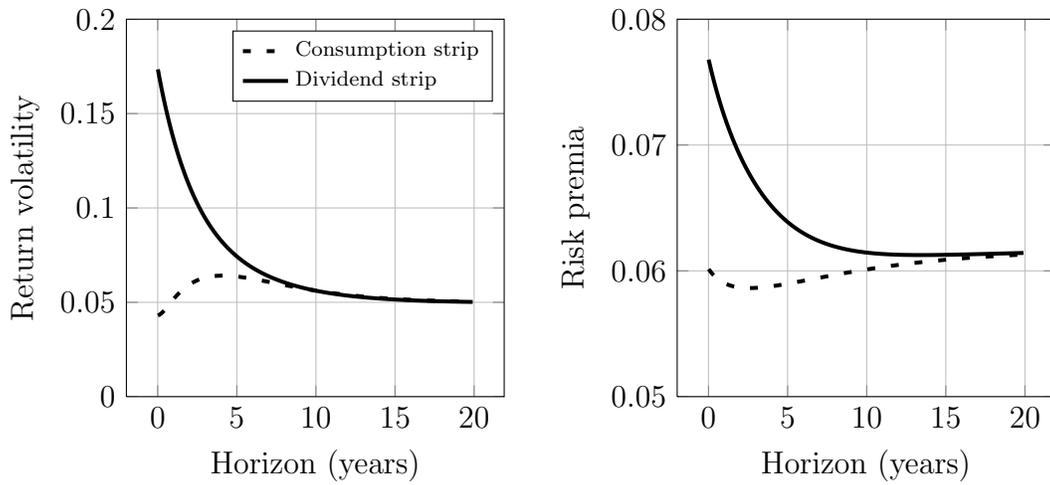


Fig. 4 Term structure of return volatility and risk premia for consumption and dividend strips.

The left graphic depicts the return volatility of consumption and dividend strips for horizons ranging from 0 to 20 years. The right graphic depicts the corresponding term structure of risk premia. State variables are $x_t = z_{1t} = z_{2t} = 0$, $\lambda_t = \bar{\lambda}$, and $\mu_t = \bar{\mu}$, and the calibration is provided in Table 3.

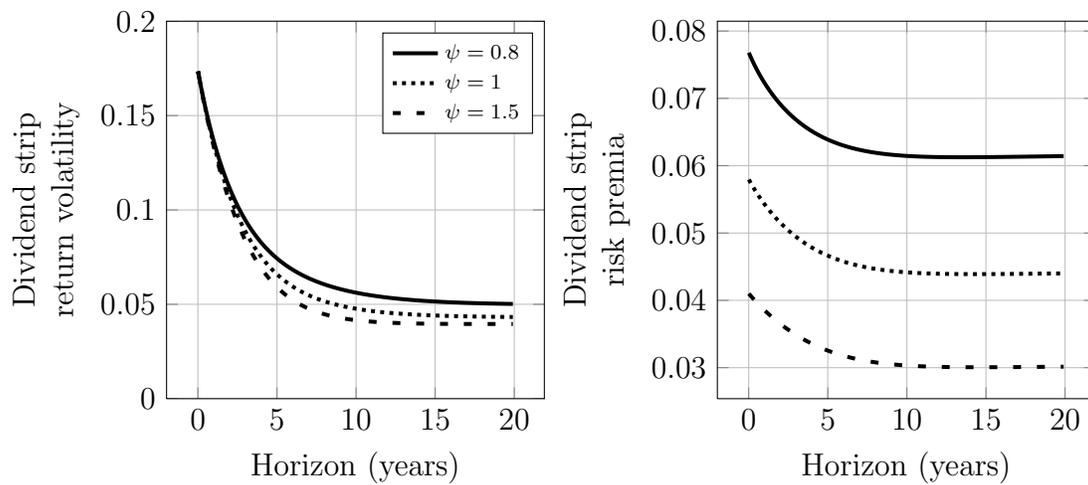


Fig. 5 Term structure of equity return volatility and risk premia versus elasticity of intertemporal substitution.

The left graphic depicts the term structure of equity return volatility for horizons ranging from 0 to 20 years. The right graphic depicts the term structure of equity risk premia. Unless stated otherwise, state variables are $x_t = z_{1t} = z_{2t} = 0$, $\lambda_t = \bar{\lambda}$, and $\mu_t = \bar{\mu}$, and the calibration is provided in Table 3.

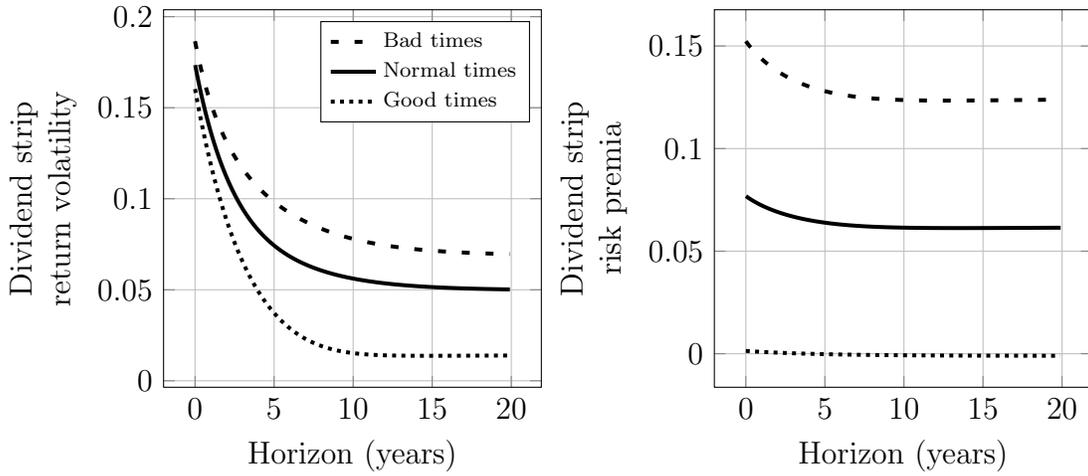


Fig. 6 Term structure of equity return volatility and risk premia versus economic conditions.

The left graphic depicts the term structure of equity return volatility for horizons ranging from 0 to 20 years. The right graphic depicts the term structure of equity risk premia. Good, normal, and bad times correspond to a jump intensity $\lambda_t = \bar{\lambda} - 0.0275$, $\lambda_t = \bar{\lambda}$, and $\lambda_t = \bar{\lambda} + 0.0275$. Unless stated otherwise, state variables are $x_t = z_{1t} = z_{2t} = 0$, $\lambda_t = \bar{\lambda}$, and $\mu_t = \bar{\mu}$, and the calibration is provided in Table 3.

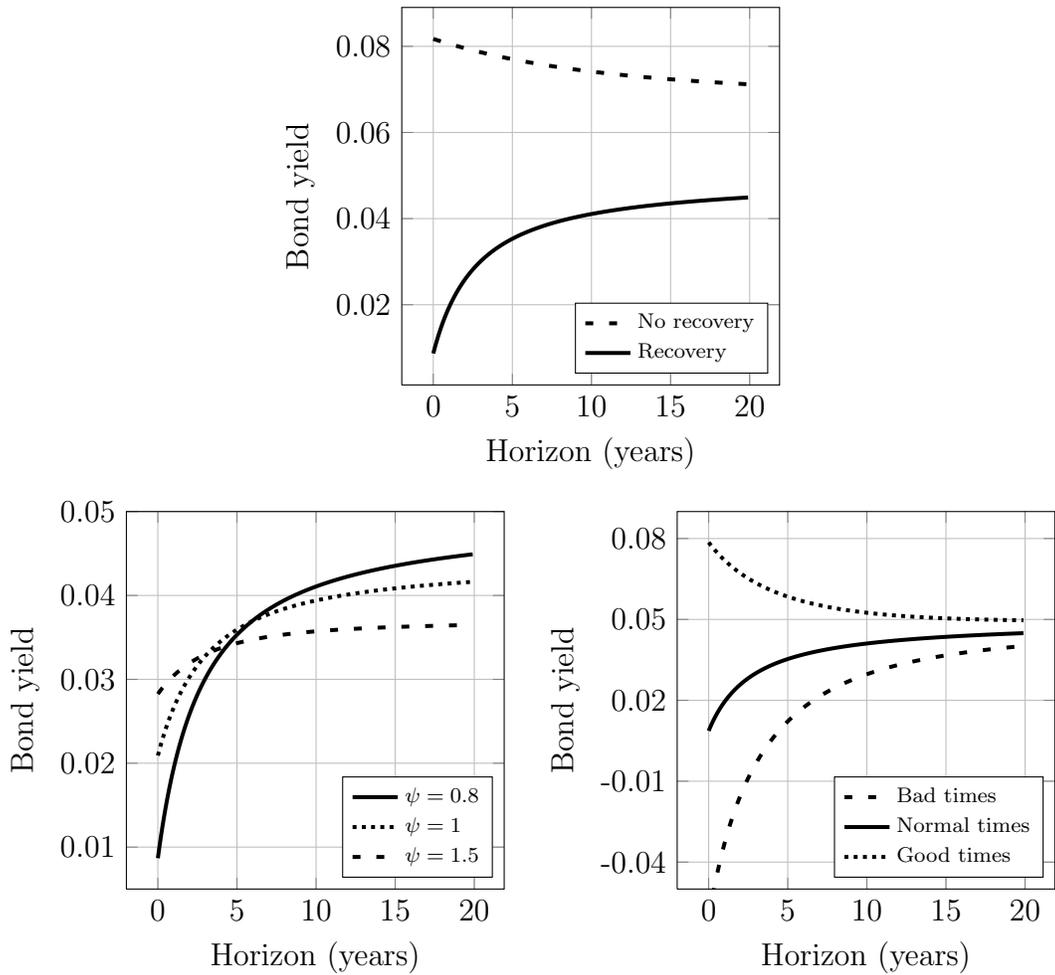


Fig. 7 Term structure of real interest rates.

The upper graphic depicts the term structure of real interest rates with ($\phi_z = 0.302$) and without ($\phi_z = 0$) recoveries for horizons ranging from 0 to 20 years. The lower left graphic depicts the term structure of interest rates for various elasticities of intertemporal substitution. The lower right graphic depicts the term structure of interest rates for various economic conditions. Good, normal, and bad times correspond to a jump intensity $\lambda_t = \bar{\lambda} - 0.0275$, $\lambda_t = \bar{\lambda}$, and $\lambda_t = \bar{\lambda} + 0.0275$. Unless stated otherwise, state variables are $x_t = z_{1t} = z_{2t} = 0$, $\lambda_t = \bar{\lambda}$, and $\mu_t = \bar{\mu}$, and the calibration is provided in Table 3.

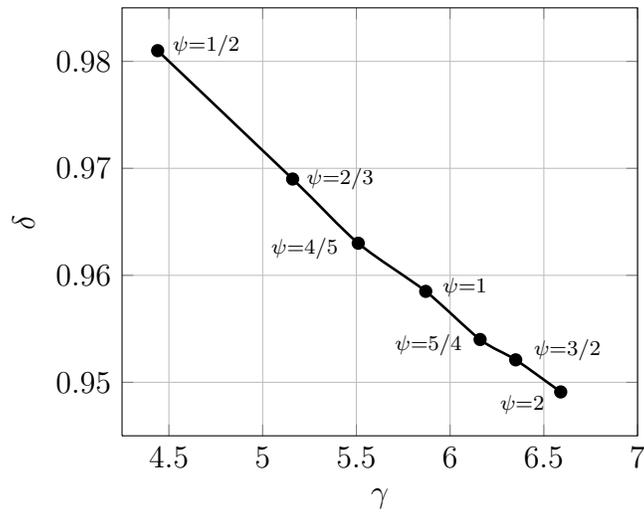


Fig. 8 Risk-free rate and risk premia versus preference parameters: robustness. The figure displays the pairs (γ, δ) required by the model to match the risk-free rate $r_t = 0.6\%$ and risk premia $RP_t = 6.5\%$ for each $\psi \in (1/2, 2)$. Unless stated otherwise, state variables are $x_t = z_{1t} = z_{2t} = 0$, $\lambda_t = \bar{\lambda}$, and $\mu_t = \bar{\mu}$, and the calibration is provided in Table 3.

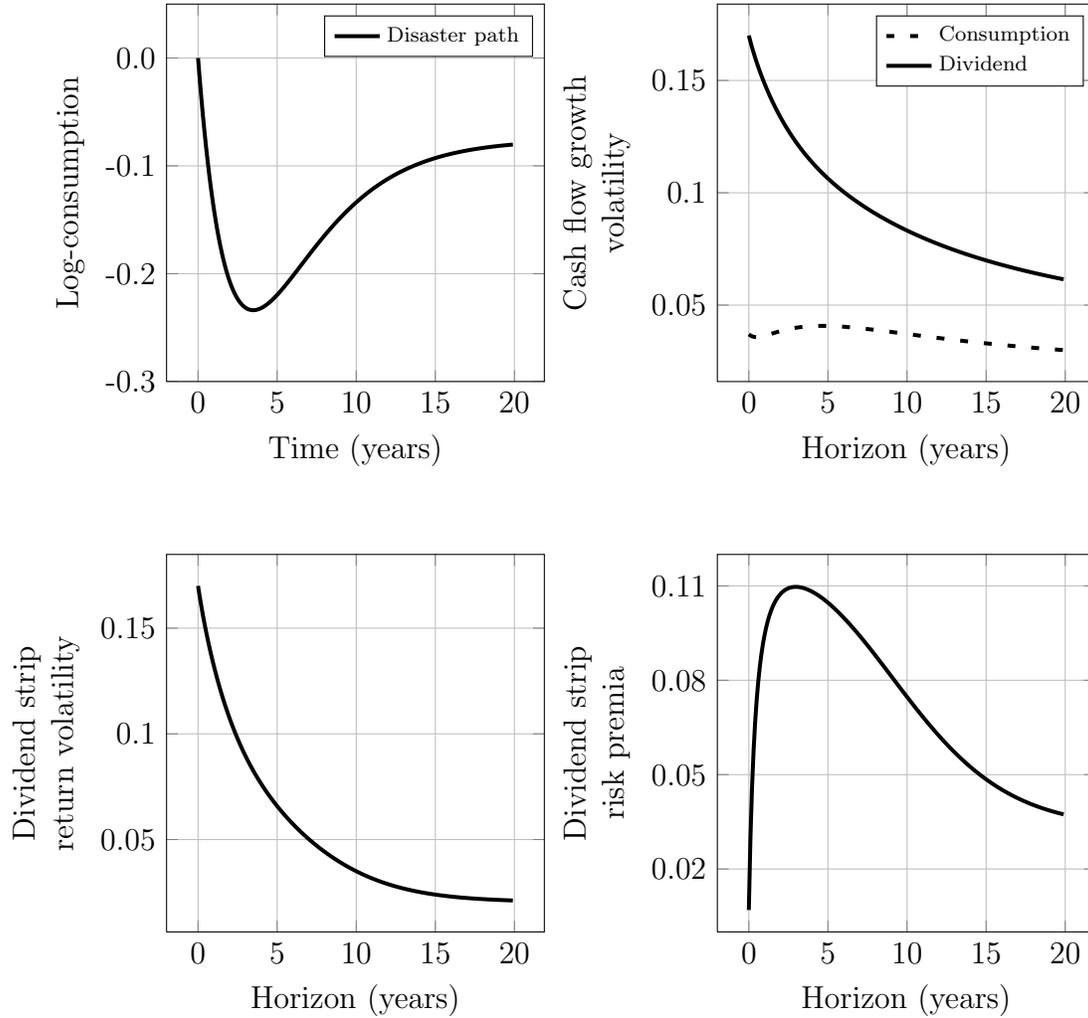


Fig. 9 Term structure of cash flow growth volatility, equity return volatility, and equity risk premia with slow disasters.

The upper left graphic reports the deterministic path of log-consumption (with no long-run growth) when a disaster occurs. The upper right graphic shows the volatility of consumption and dividend growth for horizons ranging from 0 to 20 years. The lower left and bottom right graphics depict the term structures of equity return volatility and risk premia, respectively. State variables are $x_t = z_{1t} = z_{2t} = s_{1t} = s_{2t} = 0$, and the calibration is provided in Table F1.

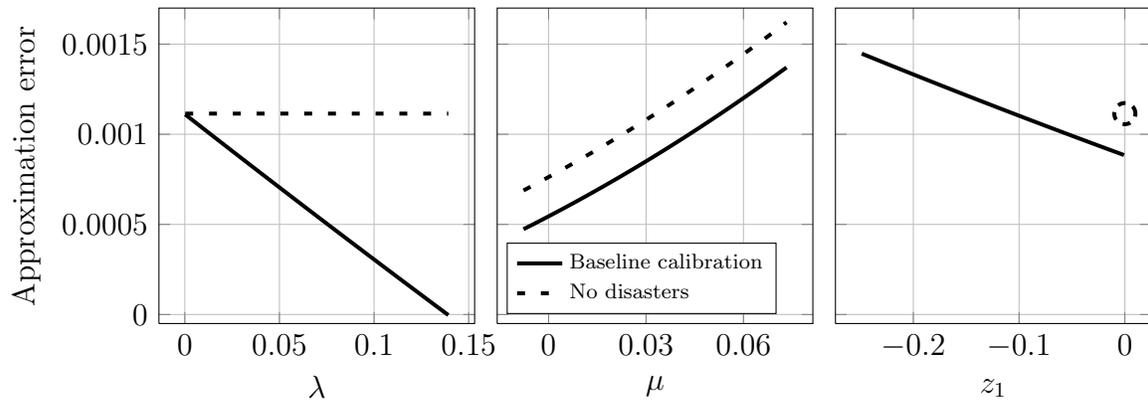


Fig. E1 Effect of disasters on the approximation accuracy.

The left, middle, and right graphics depict the partial differential equation residuals (as a fraction of $M_t V_t$) as a function of λ_t , μ_t , and z_{1t} , respectively. The solid and dashed lines are obtained with the baseline calibration and without disasters ($\eta \rightarrow \infty$), respectively. The range for λ_t and μ_t is the corresponding 99% confidence interval and that for z_{1t} is $(\mathbb{E}(\xi_t), 0)$. Unless stated otherwise, state variables are $x_t = z_{1t} = 0$, $\lambda_t = \bar{\lambda}$, and $\mu_t = \bar{\mu}$, and the calibration is provided in Table 3.