Competition and dynamics of takeover contests

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Abstract

This paper investigates the effect of potential competition on takeovers which we model as a bargaining game with alternating offers where calling an auction represents an outside option for each bidder at each stage of the game. The model aims to answer three main questions: who wins the takeover? when? and how?

Our results are able to explain why the takeover premium resulting from a negotiated deal is not significantly different from that resulting from an auction, and why tender offers are rarely observed in reality.

Furthermore, the model allows us to draw conclusions on how other dimensions of the takeover process, such as termination fees, target resistance and tender offer costs, affect its dynamics and outcome.

Keywords: takeover negotiations, auctions, bargaining, outside option.

JEL Classification: G34, C78.

1 Introduction

Since the seminal paper of Manne (1965) changes in corporate control have been considered a key mechanism in corporate governance. Potential challenges to control discipline the incumbent management only to the extent that efficiency-improving raiders compete for control of firms with weak internal governance. A vast empirical literature tries to assess whether this theory is at work in reality reaching controversial conclusions. For example, Moeller et al. (2007) find that less than four percent of deals are subject to public competition and Betton et al. (2008) report that 95 percent of the takeover bids considered in their sample are single-bid contests. On the other hand, Aktas et al. (2010) provide compelling evidence of strong latent competition during the private takeover
process and show that more potential bidders are associated with higher takeover bids.\footnote{In the recent Kraft-Cadbury takeover for instance, Kraft finally raised its bid when a counteroffer by Hershey became likely, although in the end this offer did not materialize. In another high profile case, the takeover of EMI by the private equity company Terra Firma, Terra Firma was induced to raise its bid for fear of facing an auction although no competing offer was ever made.} These latter empirical results suggest that in order to capture the role of potential competition in takeover processes, it is more appropriate to model takeover negotiations as proper “contests” where any initial bids may attract competition from rival bidders (Betton and Eckbo (2008)).\footnote{“Perhaps the most straightforward way to advance our understanding of aggregate merger activity is to model the takeover process from basic, microeconomic principles”, Betton and Eckbo (2008, p. 403).}

Building on these observations, our paper aims to investigate the effect of potential competition on the negotiation dynamics as well as on the outcomes of takeover contests. We analyze takeovers initiated by acquiring companies with unsolicited offers and we model the takeover negotiation as a bargaining game where both parties have access to an outside option at each stage: namely, the possibility to call a private auction (for the target) or a tender offer (for the raider).

The purpose of our analysis is to answer three key questions. \textit{Who wins the takeover?} We characterize under which condition the initial bidder is able to secure the deal rather than losing it to a competitor. \textit{When?} We study the duration of the takeover process. And finally, \textit{How?} We investigate the mode of completion, i.e. whether the takeover ends with agreed deal or with auction. Our model also allows us to draw conclusions on the size of the takeover premium in negotiated deals and in auctions as well as to assess the impact of other features of the takeover contest - termination fees, maximum length of the negotiation, information costs to enter an auction and tender-offer costs - on the contest dynamics and its outcome.

To address our research questions, we build a bargaining model of alternating offers over a finite horizon that is an adaptation of the Rubinstein-Stähl game over an infinite horizon (see Rubinstein (1982); Shaked (1994); Sloof (2004)). The bargaining process develops as follows. A raider starts the negotiations by making an unsolicited offer for the target.\footnote{Aktas et al. (2011) provide evidence that in their sample only 19 percent of negotiated deals are initiated by the target.} The incumbent, who controls the target firm and enjoys private benefits of control that would be lost if the raider succeeds, has three possible strategies to respond to the raider’s offer: he can accept the offer in which case the deal is agreed upon and the game ends; he can opt-out and call for a private auction; or he can reject the offer but continue the negotiations making a counter offer to the raider. In this latter case, the raider can choose among the same strategies with the only difference that opting out implies that the raider makes a tender offer, as announced sometimes in a "bear hug letter" (Betton and Eckbo (2008); Boone and Mulherin (2009)). Conversely, if the raider stays in the negotiation, the process continues to the next round where the incumbent moves first making an offer to the raider. In
this new round the players’ strategies are the same as in the previous one only with reversed roles. The parties can keep negotiating until a finite deadline is reached. Thus, in our model both parties can opt-out from the negotiation at each stage exerting their respective outside options, a private auction or a tender offer. We model these outside options as multi-stage auctions where competing bidders have to pay an information cost to learn their synergy with the target and then decide whether to enter the auction or not. Hence, the key feature of the model which innovates relative to the existing literature is that auctions and negotiations are interrelated and not mutually exclusive.

Our analysis generates numerous interesting results. Firstly, our model is able to explain why observed takeover premia in negotiated deals and private auctions are not statistically different. This is consistent with the empirical evidence documented by Boone and Mulherin (2007). This arises from the fact that the premium in negotiated deals incorporates the payoff from going to an auction. We also show that the takeover premium increases with potential competition but it decreases with the information cost potential competing bidders have to pay to enter the auction. Also, our model allows us to disentangle the effects of target resistance from the effects of potential competition on the takeover outcome. We find that the takeover premium is not affected by the level of target resistance, proxied by the benefits of control for the incumbent, whenever the raider chooses preemptive bidding in order to deter the entry of competing bidders (in line with Fishman (1988)). Otherwise, if the entry of potential competitors is not preempted, high control benefits increase the takeover premium as in the target resistance theory (Bebchuk (1994)). Our results differ from Dimopoulos and Sacchetto (2011), who also investigate the impact of preemptive bidding relative to that of target resistance, as we find that potential competition is the key driver of takeover premia whereas target resistance plays a role only for relatively weak bidders.

Furthermore, our results suggest that most deals are negotiated and are concluded at the first round of negotiation. The two parties anticipate the value of their credible threats in the negotiation and, whenever possible, reach an agreement as soon as possible. This feature of our model is consistent with existing empirical evidence by Betton and Eckbo (2008) showing that the median duration of contests when firms are private is zero days; it is also consistent with the observation of few public auctions.4 With regard to the mode of completion, we find that, in equilibrium, tender offers are never observed.

Finally, we also analyze the effect of termination fees, amongst other things, on the takeover outcome and find that sufficiently low termination fees do not impair competition suggesting that the recent decision of the Takeover Panel to ban termination fees might not be in the interest of target companies contrary to their expectations.

The effect of competition on takeover dynamics is relevant from a regulatory point of view as it is at the heart of the new European Take-over Directive

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4Similarly, Aktas and de Bodt (2010) and Moeller et al. (2007) report that less than 4% of deals are subject to observable public competition.
The stated objective of the Directive is to create a "free and open market for corporate control [and] a level playing field where market forces will determine the outcome of a takeover contest" (Ferrarini and Miller (2009)). In order to achieve this objective, the rules set by the directive require the target board to search for alternative and competing bids but also to remain passive and not to engage in any defensive strategies during the takeover contest. Similarly, the recent amendments to the Takeover Code in the UK aim to enable the target of a takeover bid to create enough competition in order to maximize shareholder value (The Takeover Panel Code Committee (2011)).

The reminder of the paper is organized as follows. The next section reviews the related literature on mergers and acquisitions as well as on bargaining games. Section 3 details the model. The main results are reported in Section 4. In Section 5 we provide a discussion of the results and their main empirical predictions. Section 6 concludes. All proofs are collected in the Appendix.

2 Literature review

Our paper contributes to the literature on takeover contests and to the bargaining literature which are reviewed below.

2.1 Takeover Literature

The idea of modeling merger negotiations as processes that embed an auction is not entirely new to the takeover literature. To the best of our knowledge, however, most of the papers compare auctions with one-to-one negotiations for the purpose of identifying the most efficient sale mechanism.

Berkovitch and Khanna (1991) do compare auctions with negotiations but consider the two takeover mechanisms as mutually exclusive. In their model, bidders, who enter the game sequentially and have to learn their synergies with the target, choose between negotiating the deal with the target or calling a tender offer at the beginning of the process. Thus, auctions do not represent an outside option in the bargaining process as in our paper and the bidders cannot switch from negotiations to auctions during the process. In this context, Berkovitch and Khanna (1991) find that bidders who discover to have low synergies with the target never choose tender offers because the potential competition is too strong.

Bulow and Klemperer (2009) also develop a model where a seller can choose between two mutually exclusive sale mechanisms, an auction or a sequential sale.

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5 This is implemented by the board neutrality principle during takeover contests and the use of so called breakthrough rules. The use of breakthrough rules aims at invalidating a variety of defensive tactics such as poison pills, dual-class shares structure, limitations on voting rights that can result in the frustration of the takeover bid. Our model is consistent with this regulation because we exclude any defensive tactics except the quest of competitive bidders.
They model the auction as a standard English auction whereas the sequential sale mechanism is such that each bidder can choose whether to enter or not, and if so the bid to make (similarly to our multi-stage auction). They show that ascending auctions are more profitable than sequential sales because they encourage more bidders to enter. The result is driven by the fact that in a sequential sale bidders can use preemptive bidding to prevent the entry of other bidders. The optimality of preemptive bidding in a sequential auction was first shown by Fishman (1988), (1989). He proved that bidders with high private valuations of the target can optimally decide to offer a high premium at their very first bid in order to signal their high valuation to potential bidders, and henceforth discourage their entry.

Dimopoulos and Sacchetto (2011) extend Fishman’s model with the purpose of disentangling the effects of preemptive bidding and target resistance on the takeover premia. In their model, target resistance is modeled as an exogenous minimum offer set by target shareholders. This reservation price is learned by the winning bidder only at the end of the bidding process and, at that point, if the winning bid is below the minimum bid the bidder has the possibility to raise the offer. In their empirical test, Dimopoulos and Sacchetto (2011) provide evidence that the high premia observed in single bidder contests result from the need to overcome target resistance rather than potential competition. Our model differs from Dimopoulos and Sacchetto (2011) in that we endogenize this minimum acceptable offer, which, in each stage of the negotiation, equals the value of the target shareholders’ best credible threat. As a consequence we obtain different predictions about the impact of potential competition relative to target resistance on takeover premia.

Povel and Singh (2006) also study the optimal mechanism for selling a firm. In their model, bidders are not equally informed about the target value. This implies that bidders with a less precise estimate of the target value are more concerned about the winner’s curse and thus bid less. In this context, the authors show that a sequential procedure in which the seller starts communicating exclusively with the best informed bidder is optimal.

A more recent strand of the takeover literature has devoted a lot of attention to the private process that takes place prior to the public announcement of a bid. In their seminal paper, Boone and Mulherin (2009) suggest that a takeover process may involve up to eleven steps and entails substantial competition, even though auctions with many bidders are rarely observed in reality. Building on these observations, Betton et al. (2008), (2009) and Aktas et al. (2010) suggest that takeover negotiations should be assumed to be conducted "under the shadow of auctions" (Eckbo (2009)).

Betton et al. (2009) develop a model of merger negotiations followed by an open auction where the initial bidder competes against a single competitor. This framework is then used to investigate the initial bidder’s optimal toehold strategy. The main difference with respect to our paper is that in Betton et al. (2009) merger negotiations always end up in an auction whereas we model auctions as an outside option for both the raider and the target, hence they represent only one possible outcome of the takeover process. Aktas et al. (2010)
instead construct an empirical measure of the degree of potential competition based on the idea that merger negotiations explicitly take place "under the shadow of an auction". They show that the credible threat of an auction during negotiations may explain why the bid premia in negotiated deals are statistically identical to those in auctions, a result first pointed out by Boone and Mulherin (2007). Aktas and de Bodt (2010) also look at the market reaction to mergers announcements and find that target stock prices react in the same way to auctions and to negotiations as documented also by Boone and Mulherin (2007). Consistent with these findings, our results show that the bid premium agreed in the negotiation anticipates the expected outcome of a potential auction. This in turn implies that high initial premia may be due to the fact that the raider anticipates high potential competition rather than to preemptive bidding.

Finally, although it is not the main focus of the analysis, our paper draws some conclusions on the role of termination fees. Several papers have investigated the effect of termination provisions, such as inducement fees, in takeovers but reach different conclusions. Coates and Subramanian (2000) and Bates and Lemmon (2003) find that termination fees adversely affect competition and ultimately prevent allocative efficiency. Officer (2003), however, claims that the apparent negative impact of termination provisions on competition is in fact due to other deal characteristics. More recently, Boone and Mulherin (2007) show that termination provisions, i.e. fees and stock option agreements, increase takeover competition in the sense that they effectively compensate a bidder in the event that the target is ultimately acquired by a competitor. Our results are in line with that of Boone and Mulherin and suggest that the effect of termination fees depends on their size.

### 2.2 Bargaining Literature

Our model develops an extension of the traditional Rubinstein bargaining game with alternating offers (Rubinstein (1982)). A specific feature of this game is that it always generates an efficient equilibrium with no delay, that is where parties find an agreement immediately. Much of the subsequent bargaining literature has tried to modify the basic Rubinstein game in order to account for more realistic features of bargaining procedures such as rejected offers, agreements near the deadline or no-agreement outcomes. In a seminal paper, Ma and Manove (1993) extend the standard Rubinstein bargaining game by introducing uncertainty about the players' strategies. Specifically, they assume that (a) players can strategically delay their offers without losing their turn; and (b) each offer is transmitted to the other player with a random delay. In this context, their model generates, like ours, equilibria where the agreement is reached at the end of the game. However, in our model the the source of uncertainty that leads the result concerns the possible entry of a stronger competitor after the first bid is made. This in turn implies that, while in Ma and Manove (1993) both parties are better off delaying the agreement to a later stage, in our model only the target has an incentive to delay the agreement beyond the first stage.

In the same spirit, Perry and Reny (1993) develop a modification of the
Rubinstein alternating offer game by imposing the following two restrictions on the players’ strategies: (a) each player has to wait a positive amount of time, a "waiting time", before being able to submit a new offer; and (b) each player can respond to his opponent only after a given (non negative) amount of time called the "reaction time". The model then generates equilibria with delay whereas the traditional no delay equilibrium arises when the reaction time is equal to zero.

Our paper differs from the previous ones because the uncertainty in our bargaining game stems from probability of facing a stronger competitor during the negotiation which, in turn, implies that the value of the outside option is endogenously determined and changes over time. Similarly to the papers above, our model yields a wider set of possible equilibria which include equilibria where the agreement is reached at the end of the game as well as equilibria with no agreement.

Finally, our model builds on Sloof (2004) who models a bargaining game with a finite horizon and alternating offers where both agents have outside options. Our model extends and enriches Sloof’s one in that the value of the outside option changes over time because it depends on the entry of a second bidder which is uncertain. As a result, while in finite-time bargaining games as in Sloof (2004) the players’ value of staying in the negotiation decreases as players approach the final date, in our model, the raider’s value of staying in the game actually increases as time goes by because he faces less potential competition.

3 The model

An incumbent $I$ (he hereafter) owns a controlling stake $\gamma > 0$ in the target firm. $I$ can be thought of as a large blockholder or the target management. We assume that $I$ conducts the takeover negotiations for the target throughout the process. We assume that the block $\gamma$ provides its owner with control of the target firm, which entitles the incumbent to non-transferrable private benefits $P > 0$. Small, dispersed shareholders own the remaining $1 - \gamma$ shares in the target company.\(^6\)

At $t = 1$ a second firm $R$, the raider (she hereafter), makes an unsolicited bid to $I$ for the acquisition of the company. Note that contrary to Boone and Mulherin (2007) our model focuses on takeovers that start with an unsolicited - potentially hostile - offer and can turn into a private sale or a tender offer at later stages. This enables us to account not only for competition among bidders but also for different attitudes of the bidders towards the target, i.e. hostile vs friendly. Further, the unsolicited bid implies that if the raider takes over the target, $I$ will lose his private benefit $P$. Because we do not want to focus on the strategic decision by $R$ about the optimal stake to buy in the target,

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\(^6\) As Shleifer and Vishny (1986) point out, while a large shareholder has incentives to monitor the firm closely because he internalizes a large part of the benefits generated, he will not internalize all of them, but, on the other hand, pays all the costs. This gives room for external raiders’ takeovers.
any offer by R is considered to hold for 100% of the target shares. The offer is privately negotiated with I and publicly announced and submitted to all other shareholders once an agreement is reached (see for example Hansen (2001)). We assume that whenever I tenders its stake to R at a certain price, all minority shareholders do the same. Conversely, if I does not sell to R, the minority shareholders cannot sell at the same conditions offered to I (simply because they have not received the offer yet). Notice however that our results hold also if R bids only for the block \( \gamma \), provided that she buys it from I and not from the minority shareholders.\(^7\)

At time \( t = 1 \) all firms values are normalized to zero. The (per share) value of the target firm for R is equal to \( r \in [0, 1] \) which represents the present value of the additional cash flows generated by the merger. We assume that the synergy \( r \) is only known to R,\(^8\) while it becomes known to both R and I at the beginning of their negotiation.\(^9\) In the following, all payoffs are expressed on a per-share basis. Finally, we assume that bids are paid with internal cash, the target is all-equity financed and players are all risk-neutral profit-maximizers.

The bargaining game

At \( t = 1 \) the game starts with R submitting a first unsolicited bid \( \beta_1 \) to I for the acquisition of the target company and a negotiation process starts between R and I to find an agreement over the sale.

We model the bilateral takeover negotiation as an alternating-offer bargaining game over \( T \) periods, with \( T \) even and finite, a variant of the Rubinstein-Stähle infinite alternating-offer game. We assume the negotiation has a finite horizon for two main reasons: (i) an economic reason, that is all bargaining surplus is likely to dissipate after a certain time preventing the negotiation from going on forever; (ii) a strategic reason, that is a finite horizon model is strategically non-stationary in the sense that late subgames are not equivalent to those starting in earlier periods (Ma and Manove (1993), Muthoo (1999), Sloof (2004)). Additionally, assuming a finite horizon is consistent with takeover practice and regulation in Europe. For instance in the UK, the recent amendments to the Takeover Code have imposed a 28 day deadline on the "virtual bid", i.e. the period during which the offeror has shown interest in the target without yet committing to a firm offer (Takeover Panel Code Committee, 2011).\(^10\)

\( R \) (I) makes an offer in all odd (even) periods. At each point in time, the offer consists of a price per share to be eventually paid by R if the two parties...
reach an agreement. We denote by $\beta^t$ the price offered by $R$ in odd periods and by $\beta^{t+1}$ the price asked by $I$ in even periods. Upon receiving an offer at $t$, the second player (i.e., the respondent) has three possible responses. He can either (1) accept the offer (hereafter, this strategy will be denoted as \{agree\}); (2) reject the offer and delay the bargaining process to the next period, if $t < T$ (\{stay\} hereafter); or, (3) opt-out of the negotiation process. If the respondent agrees to the proposed offer, the negotiation ends and $R$ takes over the target purchasing (at least) the block $\gamma$ of the target shares at the agreed bid. If the respondent rejects the offer and delays, the proposing party can decide at his turn whether to stay in the game and continue bargaining in the next period or to opt out of the negotiation. Finally, if the respondent rejects the offer and opts out, then the bargaining process ends. The dynamics and payoffs of the respective opt-out options for $R$ and $I$ are detailed in the next subsection.

Note that, being $T$ even, $R$ has the first-move advantage whereas $I$ has the last-move advantage if the negotiation reaches period $T$. This is the only asymmetry in the game, and it is motivated by the fact that the initial offer by $R$ is unsolicited. The last-move advantage consists of the respondent having a restricted strategy space because, at $T$, the option to reject and delay is empty and $R$ can only either accept $I$’s offer or opt out of the game.

Figure 1 illustrates the diagram of the decision nodes in periods $t = 1, 2$ and $T$.

The outside option payoffs and their dynamics

The main novelty of the model relative to the existing literature (e.g. Fishman (1988), Berkovitch and Khanna (1991), Dimopoulos and Sacchetto (2011)), is represented by the dynamics of the opt out option. We endogenize this by giving both players the possibility to call an auction at any stage if no agreement has been reached. This allows us to provide new insights into the evolution and the outcome of takeover processes and is consistent with Aktas et al. (2010) and Betton et al. (2009), who consider models of merger negotiations under the threat of an auction.

If at a certain stage $t$ of the negotiation $R$ decides to opt out, she initiates a tender offer for 100% of the target shares upon the payment of some cost $c_{to} > 0$ (hereafter we denote this strategy by \{TO\}). We model the tender offer as a multi-stage ascending (English) auction, in which other potential bidders might enter sequentially once the auction has started. (Engelbrecht-Wiggans (1988), Berkovitch and Khanna (1991)). In order to streamline the analysis we consider only one potential competitor, referred as $B$, but results are qualitatively equivalent if we allow for two potential competitors to sequentially enter the auction.

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11 $R$ would equally pay the same price to any minority shareholder who decides to tender his shares.

12 Betton et al. (2009) similarly assume that the target management starts their takeover game “accepting or rejecting an invitation by the initial bidder (B1) to negotiate a merger”. Aktas et al. (2011) show that in their sample more than 80% of the negotiated deals are initiated by the bidder.

13 We assume that the opening bid in the tender offer is not constrained by the previous offers made to $I$ during the private negotiation.
The game unfolds as a sequence of alternate offers over $T$ periods with $T$ even. At time 1, and in all odd periods, the raider ($R$) makes an unsolicited offer to the incumbent ($I$). Then, $I$ has three possible strategies: agree in which case a deal is agreed on at the proposed offer; opt-out in which case $I$ pays the raider some termination fees and starts a private auction (PA) in which $R$ might face competitors friendly towards $I$ and whose synergy are ex-ante unknown; stay that in turn gives $R$ the possibility to stay in the negotiation and launch a tender offer at her turn. At all time $t$ being an even number $I$ starts proposing an offer to $R$ and the two bargainers have the same set of strategies: $R$ can accept $I$’s offer, opt-out and go to a tender offer or delay and let $I$ decide whether to continue to negotiate or to opt-out with a private auction. All periods repeat in the same way till time $T$ where the option to delay for both players becomes empty.

Figure 1: The bargaining game in stages $t = 1, 2, \ldots T$.

provided they are ex-ante identical and their private valuations are i.i.d.\textsuperscript{14} Finally, we assume that if the competitor enters the tender offer, wins the auction and takes over the target, $I$ retains his private benefits of control in the target firm (or he is fully compensated for the loss of them). In other words, the second bidder is a friendly competitor, a sort of \textit{white knight}.

If at time $t$, $I$ decides to opt out, he can call a private auction (\{PA\} hereafter). This, however, requires him to pay some termination fees $\tau \geq 0$ to $R$.\textsuperscript{15,16} As with tender offers, we assume that there is a potential competitor $B$

\textsuperscript{14}Under these conditions, in fact, it can be shown that the bid that deters the entry of the first competitor would also deter the entry of others. Proofs of this result are available from the authors upon request.

\textsuperscript{15}Termination or inducement fees are often paid to the initial bidder if the target is ultimately acquired by another firm (Boone and Mulherin (2007)). We choose this way of modelling termination fees because in our set up the final takeover outcome is uncertain and also because in equilibrium, whenever $I$ opts out the raider $R$ loses the contest (see the solution in Section 4).

\textsuperscript{16}In the UK, Wippell and Knighton (2004) document that the most common practice is to enter termination fee agreements (TFAs) when the bid is announced or immediately beforehand, although it is not rare nowadays to observe TFAs entered at a much earlier stage. In this paper we do not model how termination fees are set. They are usually the result of negotiations between the two parties (see Rosenkranz (2005)) although the room for negotiation is often limited by regulatory constraints. For instance in the UK, until very recently termination fees were required to be \textit{de minimis} and in any case no more than 1% of the target value. The recent amendments to the Takeover Code have completely prohibited termination fees and other deal protection measures - on the grounds that they might (i) deter competing offers from making an offer [..], and (ii) lead to competing offers making an offer on less
who can subsequently enter the contest and compete against the raider.

Both in the case of a tender offer initiated by R or a private auction initiated by I, at any time t the private valuation r is known to R and I but unknown to B. The raider’s initial bid $\beta_t^1$ in any of these two contests is assumed to be public. Before observing R’s initial bid, B has prior beliefs about the raider’s valuation defined by the uniform density function $\tilde{r} \sim U[0, 1]$ which are updated upon the observation of $\beta_t^1$. The synergy $\tilde{s}$ that B can generate by acquiring the target is unknown to all participants when R proposes her first bid $\beta_t^1$ (Fishman, (1988)).

The realization of $\tilde{s}$ depends on the state of the bargaining game at time t. Specifically, let $\omega_t = \{\omega_l, \omega_h\}$ be the set of possible events occurring at each $t \in [1, T]$. For a given level of R’s private valuation r, if event $\omega_l$ occurs then $\tilde{s}$ is distributed according to a piece-wise linear distribution $F(x)$:

$$
F(x) = \Pr(\tilde{s} \leq x) = \begin{cases} 
\Phi[H_0 + (1 - H_0) \frac{x}{r}] & \text{for } x \leq r \\
\Phi + (1 - \Phi) \frac{x - r}{1 - r} & \text{for } x > r 
\end{cases}
$$

Thus, at $\omega_l$ and for a given valuation r, the realization of $\tilde{s}$ is lower (resp. higher) than r with probability $\Phi$ (resp. $1 - \Phi$), and there is a strictly positive probability $\Phi H_0$ that $\tilde{s} = 0$, while if $\tilde{s} \in [r, 1]$ then $\tilde{s}$ is uniformly distributed. Notice that before a potential competitor B enters, both R and I know that $\Pr(\tilde{s} \leq r) = \Phi$, since they know the realized r.

If instead the state of the game is $\omega_h$, then $\Pr(\tilde{s} > r) = 1$, that is B can overbid R with certainty.

The multi-stage auction evolves as follows. R submits a first bid $\beta_t^1$. Then B, having observed $\beta_t^1$, decides whether to pay a participation cost $c > 0$ to learn both its synergy $s$ and r and enter the competition. If B pays the cost c, an ascending auction starts with R and B as contestants and the bidder with the highest offer takes over the target paying the winning bid.

Figure 2 shows the structure of the multi-stage auction.

At $\omega_l$ the competitor’s decision to actually compete in the auction or not depends on R’s initial offer $\beta_t^1$ to the extent that $\beta_t^1$ contains information about the valuation r. Hence in our model, similar to Fishman (1988), R can use her initial bid to preempt the entry of potential competitor(s). Let $U(r | \beta_t^1)$ be the updated c.d.f. of $\tilde{r}$ conditional on the observed $\beta_t^1$. The expected profit for B upon observing $\beta_t^1$ is equal to

$$
\Pi_B(\beta_t^1) = E_{U(r | \beta_t^1)}[\tilde{s} - \tilde{r}] = \int_0^1 \left( (1 - \Phi) \int_r^1 (s - r)dF(s) \right) dU(r | \beta_t^1) 
$$

favorable terms than they would otherwise have done" (The Takeover Panel Code Committee, 2011). Our model is robust to the assumption of no termination fees.
The second bidder enters the auction only if $\Pi_B(\beta^t) > c$, otherwise he does not compete.

We now turn to the raider’s and the incumbent’s opt out payoffs at $\omega_l$. When they decide to opt-out, $R$ and $I$ know the true synergy level $r$ but they do not know the realization of $\tilde{s}$. Hence they conjecture that $B$ enters the auction with probability $p \in [0,1]$. Knowing that, with private, independent valuations, English auctions are equivalent to second price auctions where the unique weakly dominant strategy for both bidders is to bid their own valuation (Krishna (2010)), we can write $R$’s expected payoffs (here net of the tender offer costs $c_{to}$) given his initial bid $\beta^t$ as

$$\Pi_R(\beta^t; r, \omega_l) = \begin{cases} p \left\{ \Phi \left[ H_0 (r - \beta^t) + (1 - H_0) \left( \int_0^{\beta^t} (r - s)dF(s) + \int_{\beta^t}^r (r - s)dF(s) \right) \right] \right\} + \\ + (1 - p)(r - \beta^t) \end{cases}$$

where the first term represents $R$’s expected profit if $B$ enters the auction. In that case, $\Pr(\tilde{s} \leq r) = \Phi$. If $s = 0$ (what occurs with probability $H_0$), then the highest offer is $\beta^t_1$, while if $s \in ]0, r]$, the winning bid can be either $\beta^t_1$ (when $s \in ]0, \beta^t_1]$) or $s$ (when $s \in ]\beta^t_1, r]$). The second term defines $R$’s expected profit if $B$ does not compete, in which case $R$ pays her initial bid $\beta^t_1$.

Finally $I$’s expected payoffs from the opt out strategy when $R$ with synergy
r initially offers $\beta_1^t$ is equal to:

$$\Pi_I(\beta_1^t; r, \omega_l) = p \left\{ \Phi(1 - H_0) \left( \int_0^{\beta_1^t} \beta_1^t dF(s) + \int_{\beta_1^t}^r sdF(s) \right) + (1 - \Phi) \left( r + \frac{P}{\gamma} \right) \right\} + (1-p)\beta_1^t$$

(4)

If $B$ enters the auction but $R$ wins, then $I$ receives a price equal to $\max\{\beta_1^t, s\}$. If instead $B$ wins the auction, then $I$ receives a price $r$ ($R$’s synergy) and also keeps the control benefit $P/\gamma$. Finally, if $B$ does not enter the auction, $R$ wins the contest and pays the initial bid $\beta_1^t$.

In the rest of the analysis where applicable we simplify the notation as follows:

$$\pi_R = \Pi_R(\beta_1^t; r, \omega_l)$$
$$\pi_I = \Pi_I(\beta_1^t; r, \omega_l)$$

(5) (6)

with both expected payoffs taken net of the tender offer cost and termination fees.

The dynamics of the opt out payoffs depend on the subsequent realizations of the events $\omega_l$ and $\omega_h$ that define the state of the game. Formally, at any stage $t$, we define the history of events as the sequence of all events $\{\omega_j\}_{j=1}^t$, $\omega_k = \{\omega_1, \omega_h\}$ that occur from stage one until stage $t$. At stage $t$, both $R$ and $I$ know the history of events. If for any given $t \leq T$ all past events $\{\omega_j\}_{j=1}^t$ are equal to $\omega_1$ then the state of the game at time $t$ is $S^t = \{\omega_1\}$; otherwise, if the history contains at least one event $\omega_h$, then $S^t = \{\omega_h\}$. Given the current state of the game, we assume that events evolve according to the following dynamics:

$$\Pr(\omega_{t+1} = \omega_1 | S^t = \{\omega_1\}) = e^{-\lambda}$$
$$\Pr(\omega_{t+1} = \omega_h | S^t = \{\omega_1\}) = 1 - e^{-\lambda}$$
$$\Pr(\omega_{t+1} = \omega_1 | S^t = \{\omega_h\}) = 0$$
$$\Pr(\omega_{t+1} = \omega_h | S^t = \{\omega_h\}) = 1$$

(7) (8) (9) (10)

In other words, if it becomes known that a bidder with higher valuation $s > r$ exists and is willing compete in the auction, (i.e. the current state is $S^t = \{\omega_1\}$), this holds true throughout the rest of the game. Intuitively, competitors stronger than $R$ do not leave the game. If at $t$ there is uncertainty regarding the presence of a competing bidder stronger than $R$, (i.e. the current state of the game is $S^t = \{\omega_1\}$), then the entry of such a bidder at each period is i.i.d. and follows a negative exponential random process with intensity $\lambda$. Thus, if $S^t = \{\omega_1\}$ and the bargaining process reaches period $t + \Delta$, the probability that a bidder with valuation $s > r$ appears in the time interval $[t, t + \Delta]$ is equal to $1 - e^{-\lambda\Delta}$ for $\Delta = 1, 2, \ldots T - t$. These dynamics capture the possibility that $I$ finds a stronger and more friendly acquirer while negotiating the terms of the merger with $R$. Further, to make the analysis non trivial, we assume that $S^1 = \{\omega_1\}$. 

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The table below summarizes the payoffs of $R$ and $I$ for any $t$ and $S^t$ from opting out and calling an auction:

<table>
<thead>
<tr>
<th>Payoff</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pi_R^t(\omega; I {\text{stay}, R {TO}) = \pi_R - c_{to}$</td>
<td>$R$'s payoff upon observing $\omega$ and $I$ opting out</td>
</tr>
<tr>
<td>$\Pi_I^t(\omega; I {\text{stay}, R {TO}) = \pi_I$</td>
<td>$I$'s payoff upon observing $\omega$ and $R$ opting out</td>
</tr>
<tr>
<td>$\Pi_R^t(\omega; I {\text{stay}, R {TO}) = -c_{to}$</td>
<td>$R$'s payoff upon observing $\omega$ and $I$ opting out</td>
</tr>
<tr>
<td>$\Pi_I^t(\omega; I {\text{stay}, R {TO}) = r + \frac{\omega}{\gamma}$</td>
<td>$I$'s payoff upon observing $\omega$ and $R$ opting out</td>
</tr>
</tbody>
</table>

Payoffs (11) include the direct cost of a tender offer $c_{to}$ and the termination fees $\tau$. Also, notice that due to the stationarity of the dynamics (7)-(10), all the opt-out payoffs are constant over time.

4 The solution of the takeover negotiation

In this section we characterize the unique equilibrium of the bargaining game described above and discuss its main properties.

As it is standard in bargaining games (see e.g. Sloof (2004)), a necessary condition to reach an agreement is that the joint opt-out payoffs of the two parties is lower than the total synergy generated by the deal. In what follows, we check that this condition holds whenever necessary.

4.1 The characterization of outside options payoffs

Suppose that the bargaining game has reached state $S^t = \omega$ and either $R$ or $I$ opt out: a multi-stage auction as described in Fig. 2 starts. The pure strategies in this auction are the following\(^{17}\): $R$’s initial decision determines the first bid in the auction, $\beta_1^R$, possibly depending on her valuation $r$. $B$’s strategy $p$ specifies the probability that he pays the entry cost $c$ and then competes ($p = 1$) or not ($p = 0$), conditional on the raider’s initial offer $\beta_1^I$. Let the conditional density function $U(\tilde{r} | \beta_1^I)$ denote $B$’s ex-post beliefs over the distribution of the synergy $\tilde{r}$ conditional on $\beta_1^I$. $B$ competes if his expected profit from entry (4) outweighs the entry cost $c$. A Perfect Bayesian equilibrium (PBE) in the multi-stage auction is such that (i) $\beta_1^I$ maximizes (3) given the competitor’s decision to enter or not; (ii) the entry decision $p$ is rational in the sense explained above given beliefs $U(\tilde{r} | \beta_1^I)$ and (iii) $U(\tilde{r} | \beta_1^I)$ is consistent with the initial distribution $\tilde{r} \sim U[0,1]$ upon observing $\beta_1^I$.

Our multi-stage auction has a unique equilibrium whose structure is analogous to that in Fishman (1988). Raiders with valuation $r$ higher than a threshold $\tau \in [0,1]$ offer an initial preemptive bid $\beta_1^I$, denoted as $\gamma$ in the following, high enough to signal that $r \geq \tau$ where $\tau$ is the minimum valuation that deters $B$ from competing in the auction. Formally, the threshold $\tau$ is such that $\Pi_B(\beta_1^I = c$ where $U(\tilde{r} | \beta_1^I)$ is defined over the interval $[\tau,1]$. On the contrary, raiders with

\(^{17}\)In order to be able to do some comparative statics, we only focus on pure strategy equilibria.
a lower synergy $r < \tau$ offer $\beta^* = 0$ which in turn signals $r < \tau$ and triggers the entry of $B$ into the auction. Formally, given the threshold $\tau$, the preemptive bid $\overline{\beta}$ then satisfies:

\begin{align}
\tau - \overline{\beta} & \geq \Pi_R(0; r, \omega_l) \quad \text{for all } r \geq \tau \quad (12) \\
\tau - \overline{\beta} & < \Pi_R(0; r, \omega_l) \quad \text{for all } r < \tau \quad (13)
\end{align}

The Proposition below provides a formal characterization of the equilibrium:  

**Proposition 1**: Assume $c < \frac{1 - \Phi}{\Phi}$, the game has reached state $S^t = \{\omega_l\}$ and either I or R opt out. Then, for any $t \in [1, T]$ there exists a unique Perfect Bayesian equilibrium in the multi-stage auction such that:

(i) $R$ with synergy $r \geq \tau = 1 - \left(\frac{4\Phi}{\tau - \Phi}\right)$ makes an initial bid

$$\overline{\beta} = \tau \left(1 - \frac{\Phi}{2} (1 + H_0)\right)$$

(ii) $R$ with synergy $r < \tau$ offers an initial bid $\beta = 0$, $B$’s ex post beliefs are such that $r \in [0, \tau]$ and $p = 1$. The expected payoffs of $R$ and $I$ are equal to:

\begin{align}
\Pi_R(\overline{\beta}; r, \omega_l) &= r - \overline{\beta} \quad (15) \\
\Pi_I(\overline{\beta}; r, \omega_l) &= \overline{\beta} \quad (16)
\end{align}

(iii) $R$ with synergy $r < \tau$ offers an initial bid $\beta = 0$, $B$’s ex post beliefs are such that $r \in [0, \tau]$ and $p = 1$. The expected payoffs of $R$ and $I$ are equal to:

\begin{align}
\Pi_R(0; r, \omega_l) &= r(1 + H_0) \frac{\Phi}{2} \quad (17) \\
\Pi_I(0; r, \omega_l) &= (1 - \Phi) \left(r + \frac{P}{\gamma}\right) + \Phi (1 - H_0) \frac{P}{\gamma} \quad (18)
\end{align}

It is worth noticing that the payoffs (15)-(16) and (17)-(18) are the expected payoffs of the two parties given that one of the them opts out. Once the auction starts, raiders with low valuations, i.e. with $r < \tau$, cannot deter the entry of potential competitors because they would be worse off choosing a preemptive bid since by construction $r(1 + H_0) \frac{\Phi}{2} > r - \overline{\beta}$ for all $r < \tau$. This behavior is anticipated and incorporated to compute the agreement value in the negotiation.

In order to rule out uninteresting situations we need to impose the following restrictions on the parameters $c_o$ and $\tau$:

Assumption 1 (A1): $c < \frac{1 - \Phi}{\Phi}$ and $c_o \in [(1 - \Phi) \frac{P}{\gamma} - (1 - H_0) \frac{\Phi}{2} r; \frac{\Phi}{2} (1 + H_0)]$

The restriction on the entry cost $c$ derives from Proposition 1 whereas the restriction on the tender offer cost $c_o$ is implied by the following two conditions: (1) $\pi_R(\omega_l) + \pi_I(\omega_l) \leq r + c_o$ which we need to guarantee that an agreement can be reached at some stage of the game and defines the lower bound of the interval, and (2) $\pi_R(\omega_l) - c_o \geq 0$ which ensures that $R$’s threat to go to tender offer is credible and defines an upper bound on $c_o$. In the following we will also make use of following restrictions on the values of termination fees $\tau$.

Assumption 2 (A2): $\tau < \min \left\{r - \tau \left(1 - \frac{\Phi}{2} (1 + H_0)\right); \frac{\Phi}{2} (1 + H_0) r - (1 - \Phi) \frac{P}{\gamma}\right\}$.
4.2 The characterization of the equilibrium

Because state $S^t$ is known by both contestants at any stage $t$, we can characterize the unique PBE of the bargaining game by backward induction. In order to illustrate the equilibrium construction we first describe the optimal strategies for $R$ and $I$ in the last two stages $T$ and $T-1$. We assume w.l.o.g. that, whenever a player does not want to reach an agreement at a given stage he (she) bids zero. Because they will often be used in the rest of the analysis, we summarize the multi-stage auction equilibrium payoffs for $R$ and $I$ in state $S^t = \{\omega_l\}$ obtained in Proposition 1 in the table below.\(^{18}\)

\[
S^t = \{\omega_l\} \quad r < \overline{r} \quad r \geq \overline{r}
\]

\[
\pi_R = \begin{cases} 
\pi_T (r+\frac{\beta}{T}) + \Phi (1-H_0) \frac{r}{T} & \text{if } r \geq \overline{r} \\
(1-\Phi) \left(r+\frac{\beta}{T}\right) & \text{if } r < \overline{r} 
\end{cases}
\]

The next lemma characterizes the unique equilibrium in the last two periods of the game $T$ and $T-1$.

**Lemma 1:** Under A1 and A2, the unique equilibrium at stage $T$ is a strategy profile $\sigma^T = [\sigma^T(\omega_l); \sigma^T(\omega_h)]$ such that:

(i) If $r \geq \frac{P}{\gamma}$ or if $\tau < r < \frac{P}{\gamma}$

\[
\sigma^T = \begin{cases} 
\sigma^T(\omega_l) = \{\text{agree}\} \\
\sigma^T(\omega_h) = \{\beta^T(\omega_l)\} 
\end{cases}
\]

where $\beta^T(\omega_l) = r - (\pi_R - c_{to})$ and $\beta^T(\omega_h) = r$. The continuation payoffs for $R$ and $I$ at the beginning of stage $T$ are uniquely defined as:

\[
S_T^T = \{\omega_l\} : \begin{cases} 
\Pi_T^R(r, \omega_l; \sigma^{T-1}) = r - \beta^T(\omega_l) = \pi_R - c_{to} \\
\Pi_T^I(r, \omega_l; \sigma^{T-1}) = \beta^T(\omega_l) > \pi_I 
\end{cases}
\]

\[
S_T^T = \{\omega_h\} : \begin{cases} 
\Pi_T^R(r, \omega_h; \sigma^{T-1}) = 0 \\
\Pi_T^I(r, \omega_h; \sigma^{T-1}) = r 
\end{cases}
\]

The unique PBE at $T-1$ is $\sigma^{T-1} = [\sigma^{T-1}(\omega_l); \sigma^{T-1}(\omega_h)]$ such that:

\[
\sigma^{T-1}(\omega_l) = \begin{cases} 
\sigma^{T-1}_R(\omega_l) = \{\beta^{T-1}(\omega_l); \text{TO}\} \\
\sigma^{T-1}_I(\omega_l) = \{\text{agree}\} 
\end{cases}
\]

\[
\sigma^{T-1}(\omega_h) = \begin{cases} 
\sigma^{T-1}_R(\omega_h) = \{0; \text{stay}\} \\
\sigma^{T-1}_I(\omega_h) = \{PA\} 
\end{cases}
\]

\(^{18}\)In the analysis that follows, whenever $\pi_R$ and $\pi_I$ are written without specifying the range of $r$, then results hold for all $r$ (i.e. for $r < \overline{r}$ and $r \geq \overline{r}$).
where the agreement is reached at offer $\beta^{T-1}(\omega_I) = \pi_I$. The continuation payoffs for $R$ and $I$ at the beginning of stage $T - 1$ are uniquely defined as:

$$S^{T-1} = \{\omega_I\} : \begin{cases} \Pi^{T-1}_R(r, \omega_I; \sigma^{T-1}) = r - \beta^{T-1}(\omega_I) > \pi_R - c_0 \\ \Pi^{T-1}_I(r, \omega_I; \sigma^{T-1}) = \beta^{T-1}(\omega_I) = \pi_I \end{cases}$$  \tag{22}

$$S^{T-1} = \{\omega_h\} : \begin{cases} \Pi^{T-1}_R(r, \omega_h; \sigma^{T-1}) = \tau \\ \Pi^{T-1}_I(r, \omega_h; \sigma^{T-1}) = r + \frac{P}{T} - \tau \end{cases}$$  \tag{23}

(ii) If $r < \frac{P}{T}$ and $\tau \in [r, \frac{P}{T}]$, the unique equilibrium in $S^T = \{\omega_I\}$ is the same as in (i) while in $S^T = \{\omega_h\}$ $R$ simply exits the negotiations, and the continuation payoffs of $R$ and $I$ at the beginning of stage $T$ are $\Pi^{T}_R(r, \omega_h; \sigma^{T-1}) = 0$, $\Pi^{T}_I(r, \omega_h; \sigma^{T}) = \frac{P}{T}$.

The unique PBE at $T - 1$ $\sigma^{T-1} = [\sigma^{T-1}(\omega_I); \sigma^{T-1}(\omega_h)]$ is such that:

$$\sigma^{T-1}(\omega_I) = \begin{cases} \sigma^{T-1}_R(\omega_I) = \{\beta^{T-1}(\omega_I); \text{TO}\} \\ \sigma^{T-1}_I(\omega_I) = \{\text{agree}\} \end{cases}$$

$$\sigma^{T-1}(\omega_h) = \begin{cases} \sigma^{T-1}_R(\omega_h) = \{0; \text{stay}\} \\ \sigma^{T-1}_I(\omega_h) = \{\text{stay}\} \end{cases}$$

and the agreement is reached at offer $\beta^{T-1}(\omega_I) = \pi_I$. The continuation payoffs of $R$ and $I$ at the beginning of stage $T - 1$ are uniquely defined as:

$$S^{T-1} = \{\omega_I\} : \begin{cases} \Pi^{T-1}_R(r, \omega_I; \sigma^{T-1}) = r - \beta^{T-1}(\omega_I) > \pi_R - c_0 \\ \Pi^{T-1}_I(r, \omega_I; \sigma^{T-1}) = \beta^{T-1}(\omega_I) = \pi_I \end{cases}$$

$$S^{T-1} = \{\omega_h\} : \begin{cases} \Pi^{T-1}_R(r, \omega_h; \sigma^{T-1}) = 0 \\ \Pi^{T-1}_I(r, \omega_h; \sigma^{T-1}) = \frac{P}{T} \end{cases}$$

The characterization of the equilibrium in the last two stages of the game suggests that in all states $S^t = \{\omega_I\}$ the value to each player of rejecting an offer and staying in the negotiation varies across time. To prove this formally, we need to determine the "value of staying in the game" and specify how this changes over time.

Suppose the game is in $S^t = \{\omega_I\}$ with $t < T - 1$ and that $R$ expects $I$ to continue the negotiation at $t + 1$ in the case of no agreement. Intuitively, $R$ is better off at $S^{t+1} = \{\omega_I\}$ than at $S^t = \{\omega_I\}$. Indeed, as negotiation continues into the next period the chances of a stronger competitor entering the contest decrease. Thus the value of staying in the game for $R$, given that $I$ does not exit, should increase as the game approaches $T - 1$. If this is the case, then the strategy "stay" becomes more valuable to $R$ as the game approaches $T - 1$. Conversely, conditional on $R$ continuing the negotiation in the next period, intuitively $I$ is better off at $S^t = \{\omega_I\}$ than at $S^{t+1} = \{\omega_I\}$, the reason being that as the game approaches the end the chances of finding a strong competing bidder decrease. Hence, the value to $I$ of delaying the negotiations decreases as the game approaches $T - 1$.  

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These intuitions are formalized in the next Lemma.

**Lemma 2:** Let $x^t$ (resp. $y^t$) be the expected continuation payoff of $R$ (resp. $I$) in $S^t = \{\omega_t\}$ given that no agreement is reached and no player opts out until $T - 1$ provided the state of the game remains $\omega_t$ and define $\chi \equiv (T - 1) - t$.

(i) Under A1 and A2, if $r > \frac{P}{\gamma}$ then for any $t \leq T - 1$ we have:

\[
x^t(r) = e^{-\lambda x} \left( r - \beta^{T-1}(\omega_t) \right) + (1 - e^{-\lambda x}) \tau \\
y^t(r) = e^{-\lambda x} \beta^{T-1}(\omega_t) + (1 - e^{-\lambda x}) \left( r + \frac{P}{\gamma} - \tau \right)
\]

where for any given $r$, $x^t$ decreases with $\chi$ (i.e., increases with $t$) with a maximum in $x^{T-1}(r) = r - \beta^{T-1}(\omega_t) \geq \pi_R - c_{00}$, while $y^t$ increases with $\chi$ (i.e., decreases with $t$) with a minimum at $y^{T-1} = \pi_I = \beta^{T-1}(\omega_t)$. Moreover:

\[
x^t + y^t = r + (1 - e^{-\lambda x}) \frac{P}{\gamma} - r
\]

for any $\chi > 0$.

(ii) If instead $r < \frac{P}{\gamma}$ and $\tau \in [r, \frac{P}{\gamma}]$:

\[
x^t(r) = e^{-\lambda x} \left( r - \beta^{T-1}(\omega_t) \right) \\
y^t(r) = e^{-\lambda x} \beta^{T-1}(\omega_t) + (1 - e^{-\lambda x}) \frac{P}{\gamma}
\]

where for any given $r$, $x^t$ increases (resp. $y^t$ decreases) with $t$ if $r - \beta^{T-1}(\omega_t) > 0$ and $x^t + y^t > r$. Otherwise, if $r - \beta^{T-1}(\omega_t) < 0$, $x^t$ is decreasing in $t$ and negative.

Lemma 2 highlights a peculiar feature of our game. Contrary to other bargaining models such as Rubinstein (1982) and Sloof (2004), where the value of staying in the game is either constant over time or it decreases for both parties, in our model this value increases for the raider while it decreases for the incumbent as the game approaches the end. The reason for this lies in the uncertainty surrounding the entry of potential competitors at each stage which implies that the value of the outside options is time-varying. This in turn allows us to obtain a wider set of possible equilibrium outcomes.

Also, Lemma 2 characterizes $I$’s credible threat in the case of no agreement. Because $y^t$ decreases with $t$, but is always higher than $\pi_I = \beta^{T-1}(\omega_t)$, starting a private auction is never a credible threat for $I$ at any stage $S^t = \{\omega_t\}$, $t \in [1, T - 1]$ if $R$ stays in the negotiation. Indeed, by opting out and starting a private auction $I$ would get a payoff $\pi_I - \tau$ that is always lower than $y^t$, the payoff from continuing the negotiation when $R$ also continues. $I$ prefers to delay the bargaining rather than opting out in state $\omega_t$ because, by so doing, he can still hope to find a stronger potential competitor to challenge the raider in the case he decides to opt out at a later stage.

Conversely, the credibility of $R$’s outside option can change over time. In fact, calling a tender offer at state $S^t = \{\omega_t\}$ is a credible threat only if $x^t <
\[ \pi_R - c_{t_0}; \] whereas when \( x^t > \pi_R - c_{t_0} \), \( R \) is better off staying in the game if \( I \) also stays in. Given that \( x^t \) increases with \( t \) then either opting out is never a credible threat or, if it is credible at some future state \( S^t = \{ \omega_1 \} \) \( t' < T - 1 \), then it is also credible at \( t = 1 \). Notice that this result does not hold when \( r < \beta^{T-1}(\omega_1) \) as in this case \( R \) is better off exiting at \( t = 1 \) since the value of staying \( x^1 < 0 \).

Finally, Lemma 2 provides useful insights for the construction of the unique PBE of the game at all states \( S^t = \{ \omega_1 \} , \ t \in [1, T - 2] \). Specifically, Lemma 2 implies that whenever \( x^t + y^t > r \) at any stage \( t < T - 1 \) with \( S^t = \{ \omega_1 \} \), an agreement cannot be reached in equilibrium because opting out is not a credible threat for either of the players. Hence, the negotiation between \( R \) and \( I \) continues as long as the state of the game remains \( \omega_1 \).

**Proposition 2:** Let \( A1 \) and \( A2 \) be verified, \( r > P/\gamma \) and \( r > \pi_I \). Suppose the game starts at state \( S^1 = \{ \omega_1 \} \). Then under \( A1 \) and \( A2 \) the unique PBE of the bargaining game is characterized as follows:

\( (a1) \) If \( x^1 = e^{-\lambda(T-2)}(r - \pi_I) + (1 - e^{-\lambda(T-2)})\tau \leq \pi_R - c_{t_0} \), then \( R \) and \( I \) sign an agreement immediately at \( S^1 = \{ \omega_1 \} \) with a bid \( \beta^1 = \pi_I \)

and the equilibrium expected payoffs from the takeover contest are:

\[ \Pi_R(r, \omega_1) = r - \beta^1 > \pi_R - c_{t_0} \] \( \Pi_I(r, \omega_1) = \beta^1 \)

\( (a2) \) If \( x^1 > \pi_R - c_{t_0} \) and the state of the game stays at \( S^t = \{ \omega_1 \} \) for all \( t \in [1, T - 1] \), then an agreement can be reached only at \( T - 1 \) with an offer \( \beta^{T-1}(\omega_1) = \pi_I \).

If at some \( t \leq T - 1 \) the state of the game switches to \( S^t = \{ \omega_h \} \), then

\( (b1) \) If \( r > P/\gamma \), the bargaining ends with a private auction won by a competing bidder \( B \). \( I \) pays the termination fee \( \tau \) to \( R \) and the equilibrium expected payoffs from the takeover contests are:

\[ \Pi_R(r, \omega_1) = \tau(1 - e^{-\lambda(T-1)}) + (r - \beta^{T-1}(\omega_1))e^{-\lambda(T-1)} \] \( \Pi_I(r, \omega_1) = \left( r + \frac{P}{\gamma} - \tau \right)(1 - e^{-\lambda(T-1)}) + \beta^{T-1}(\omega_1)e^{-\lambda(T-1)} \)

\( (b2) \) if \( r \leq \frac{P}{\gamma} \) and \( \tau \in [r, \frac{P}{\gamma}] \), \( R \) leaves the negotiation and the equilibrium expected payoffs from the takeover contest are:

\[ \Pi_R(r, \omega_1) = (r - \beta^{T-1}(\omega_1))e^{-\lambda(T-1)} \] \( \Pi_I(r, \omega_1) = \beta^{T-1}(\omega_1)e^{-\lambda(T-1)} + \frac{P}{\gamma}(1 - e^{-\lambda(T-1)}) \)

Interestingly, Proposition 2 shows that the takeover premium paid by \( R \) equals \( I \)’s expected payoff in case of an auction. Thus, the wealth effect to

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\(^{19}\)This is also the case when termination fees are \( \tau > r - \pi_I(\omega_1) = \frac{P}{\gamma}(1 + H_0)\tau - (1 - \Phi)\frac{P}{\gamma} \) as in this case \( R \) is better off losing the multi-stage auction.
the target shareholders is equal in auctions and in negotiations. The reason for this is that in our model the resistance of the target in the negotiation is endogenously determined by the degree of competition if an auction is started. Notice that this result holds also for raiders with high synergy \( r \geq \tau \), who, in an auction, optimally choose to preempt the entry of potential competitors by offering a high bid \( \beta \). This possibility is rationally anticipated by \( R \) and \( I \) in the negotiation and determines the premium in case of agreement.

Moreover, the premium is the same regardless of when the agreement is reached, i.e. \( \beta^1 = \beta^{T-1} (\omega_t) = \pi_I \) whenever the outcome of the game is a deal between the initial bidder and \( I \).\(^{20}\) This shows that a long negotiation does not necessarily favour the target shareholders when it is commonly known that the time for reaching an agreement is limited. This is intuitive: as the end approaches, the bargaining position of the initial bidder becomes stronger.

Case (a1) of Proposition 2 is the standard no-delay equilibrium typical of Rubinstein-like bargaining games without uncertainty. Case (a2) of the proposition however is specific to our setup. While the formal proof can be found in the Appendix, we provide here an intuitive argument for this result. In any state \( S^t = \{ \omega_t \} \) \( t < T - 1 \), if \( R \) cannot credibly threaten to opt out from the negotiation, condition (26) implies that it is too expensive for her to seal an agreement with \( I \). This because in order to do so, \( R \) would need to offer \( I \) a bid at least equal to his outside option payoff \( y^t > \pi_I \). In other words, a deal would then leave \( R \) with a share of the pie equal to \( r - y^t < x^t \), the value for \( R \) to stay in the negotiation. However, when \( x^t > \pi_R - c_{to} \), \( R \) is better off continuing the negotiation although by doing so she runs the risk of possibly facing a stronger competitor and losing the contest in the future. This is because if the game reaches \( S^{T-1} = \{ \omega_0 \} \), \( R \) gets a payoff of \( r - \pi_I > \pi_R - c_{to} \), which defines \( R \)'s payoff from opting out to a tender offer immediately. Additionally, if the game switches to state \( S^{t1} = \{ \omega_h \} \), at \( t_1 \in [t, T - 1] \) \( R \) loses the auction but earns the breakup fees \( \tau \). The termination fees then work as an insurance for \( R \) against the possibility of losing the contest before \( T - 1 \), and explain the result in (a2).

5 Equilibrium properties and empirical implications

The characteristics of the equilibrium described in Proposition 2 allow us to explain several features of takeover negotiations documented in the empirical literature as well as to deliver new interesting empirical predictions. Specifically:

1. The takeover premium at which an agreement is reached is uniquely determined given \( R \)'s valuation of the target \( r \) and it equals the target’s outside

\(^{20}\)At equilibrium \( I \) calls for an auction only in state \( S = \{ \omega_h \} \) i.e. only if a stronger competitor \( B \) appears and then the takeover ends with \( B \) acquiring the target firm.
option payoff from launching a private auction:

\[
\beta^1 = \pi = \pi = \pi(1 - \frac{\Phi}{r + \Phi}) + \Phi (1 - H) \frac{r}{2} \quad \text{if } r < \pi
\]

The takeover premium in negotiated deals equals the premium in private auction sales because the latter represents the "disagreement" payoff in the bargaining and is then included in the terms of the deal. This result may explain why the observed premia in negotiated deals and private auctions are not statistically different (Boone and Mulherin (2007)). Also, the premium increases with the degree of potential competition\textsuperscript{21} which is consistent with the empirical evidence documented by Aktas et al. (2010).\textsuperscript{22}

2. In equilibrium, most of the deals are concluded in the first round of negotiation. The two parties rationally anticipate the value of their credible threats and reach an agreement immediately when this is possible. This feature of our model is consistent with Betton and Eckbo (2008) who show that the median duration of contests when firms are private is zero days. We also predict that the deals which are not concluded in the first period are completed at very late stages.

3. The results shed some light on the effects of potential competition and preemptive bidding versus target resistance on takeover outcomes. The model predicts that preemption strongly determines the takeover premium in takeovers initiated by high synergy raiders. Conversely, in takeovers with low synergy raiders the premium is highly dependent on incumbent resistance. This result contrasts with that of Dimopoulos and Sacchetto (2011) who find that target resistance is the key driver of takeover premium.

4. Our model predicts that the acquirer always earns a positive return from the deal, both when it is reached by negotiations or by an auction.\textsuperscript{23} This is consistent with Netter et al. (2011) who show that acquisition activity is wealth increasing for the acquirer's shareholders.

5. Our model also provides testable implications about the impact of the takeover length on the final outcome suggesting that constraining the time available to negotiate the deal plays in favor of the initial raider. Allowing for long negotiations helps the target since it gives him more time to find strong competitors. This has some relevant implications in the light of the recent decision by the UK Takeover Panel to shorten the "virtual bid."

---

\textsuperscript{21}The degree of potential competition in our model is inversely related to \( \Phi \) and \( H \).

\textsuperscript{22}Our result contrasts instead with Povel and Singh (2006) who do not find that the premium increases if the competitor is stronger. The reason for the different result is due to their assumption that bidders are asymmetrically informed about the target value.

\textsuperscript{23}Indeed, the premium \( \beta^1 \) is always lower than synergies \( r \) and \( \bar{s}_1, \bar{s}_2 \).
period to 28 days (The Takeover Panel Code Committee (2011)).\footnote{Indeed, \textit{ceteris paribus}, the lower \(T\), \i.e., the shorter the period available for the negotiation, the higher \(x_1\), \i.e. the more likely the opt-out threat for the initial bidder is not credible, and consequently the more likely the negotiation continues till the last period.} We suggest that this might not be in the interest of the target as expected by the Panel.

6. The takeover premium is the same irrespective of the period when the parties reach an agreement if the competitive environment does not change.

7. The higher the probability of the target manager finding a strong competing bidder at each given period of time (captured by the parameter \(\lambda\)), the lower the raider’s incentive to stay in the game and, hence, the more likely that the deal is signed immediately. Notice that \(\lambda\) can be interpreted as the ability of the target management to find alternative competitors, which in reality is likely to depend on the target’s ownership structure as well as on the target management’s personal connections.

8. The following examines the effects of the information costs \(c\) of entering the auction, the cost of launching a tender offer \(c_{to}\), and of the termination fees \(\tau\) on the takeover outcome:

   (a) The higher \(c_{to}\), the lower the raider’s opt-out payoff, hence the less likely that opting out is a credible threat for her. As a result, this makes more difficult to reach an early agreement.

   (b) A high \(c\) reduces the expected profits of potential competitors thereby making it easier for \(R\) to deter their entry. Also, the threshold \(\tau\) and the preemptive bid \(\beta\) are negatively correlated with the auctions entry costs.

   (c) Sufficiently low termination fees \(\tau\) do not increase the takeover premium so that they do not necessarily increase the ex-ante cost of a deal for the initial bidder.

   (d) Increasing \(\tau\) has a dual effect as it increases \(x_1\) but it reduces \(y_1\), making more difficult for the two parties to reach an early agreement. As long as breakup fees are not too high, they compensate the raider for the possibility of losing the contest without however preventing the target from searching for a competitor. Hence we suggest that sufficiently low termination fees should not impair competition in takeovers in contrast with the arguments proposed by the Takeover Panel Committee in the UK which has recently banned inducement fees (The Takeover Panel Code Committee (2011)).

9. In our model, tender offers are never observed in equilibrium. However, private auctions might arise, and they are more likely to be observed when \(T\) is small, \(\lambda\) is low and termination fees are (relatively) high. All these elements increase the value for \(R\) to delay the agreement, thereby allowing
I time to find stronger competitors and organize an auction. This result is consistent with the observation of few public auctions (Aktas et al. (2010) and Moeller et al. (2007)).

10. The possibility that the raider exits the game without submitting a second offer arises when it is commonly known that a potential competitor with higher valuation has entered and would win a potential auction for the target.

6 Conclusions

The interest in the specific dynamics of takeover contests has grown recently. Eckbo (2009, p. 3) points out that "in a very real sense, merger negotiations occur in the shadow of an auction, so the expected auction outcome affects the bargaining power of the negotiation parties". Along the same lines, Boone and Mulherin (2007, p. 848) stress the importance of understanding the role of what they define the "complex private takeovers process (that) evolves prior to the public announcement of a takeover bid" in order to draw conclusions on the efficiency of the market for corporate control. In a recent paper, Aktas et al. (2010) provide empirical evidence supporting the conjecture that many takeover negotiations are in fact conducted under the threat of an auction. However, despite the available empirical evidence, theory lags behind in explaining the dynamics of such takeover processes. Our paper represents a first attempt to shed light on how competition, both ex ante and ex post, affects the evolution of a takeover contest.

We build a bargaining model of alternating offers adapted from the Rubinstein-Stähl model that explicitly incorporates the possibility for each player to opt-out at any stage of the negotiation and start an auction that potentially involves a competing bidder. The model is able to generate a number of interesting results about the characteristics of the takeover outcome. Specifically, the model is able to predict the duration of the takeover process; whether it ends with a negotiated deal or with an auction; and the size of the takeover premium.

The richness and flexibility of the model allows us to capture and account for numerous important dimensions of takeover contests in practice, namely the cost of tender offer, termination fees, potential competition and the length of the negotiation.

Our results provide a theoretical rationale for some puzzling empirical facts such as why takeover premia in negotiations do not differ significantly from premia in auctions; why tender offers are rarely observed in reality; and, finally, why the duration of takeover negotiations is typically very short.

25 $R$ always makes a first bid that starts the whole process in our model.
26 This corresponds to what we define as state $\omega_h$ in our model.
References


8 Appendix (proofs)

**Proof of Proposition 1**: The characterization of the equilibrium is done in two steps: (i) we determine the minimum threshold $\tau$ such that for any $r \geq \tau$, B’s expected profit from competing is non-positive; formally, $\tau$ is such that for any $r' < \tau$, B is not deterred from competing if he knew that $r \geq r'$ while he would be deterred if he knew that $r \geq r''$, for all $r'' > \tau$; (ii) we compute the minimum bid $\beta_1$, hereafter $\beta$ for simplicity, that signals $r \geq \tau$ assuming that the best alternative to pre-emption of potential entrants is $\beta_1 = 0$.

(i) To uniquely determine the minimum threshold $\tau$ that deter B to participate to the auction, we first describe his entry choice. Suppose that upon observing a first bid $\beta$, at equilibrium $B$ updates the c.d.f. of $\tilde{r}$ to $U (x | \beta) = \frac{Pr(\tilde{r} \leq x | \beta)}{\frac{1}{\tilde{r}}} \quad \text{for} \quad r \in [\tau, 1]$. Also, when deciding whether to enter or not, $B$ does not know his valuation $\tilde{s}$, but he knows that, for any $r$, $\tilde{s} > r$ with
probability $1 - \Phi$. Given that each bidder’s weakly dominant strategy in an English auction is to bid up to its own valuation, $B$ then expects a payoff equal to $(1 - \Phi) \int_r^1 (s - r) dF(s)$ upon entrance, since $s$ is distributed according to $F(s)$ when $s \in [r, 1]$, with $F(s)$ as in (1). Integrating this value for all possible types $r$ according to $U(x | \beta)$ we obtain that the expected revenue of $B$ from entry is

$$
\Pi_B(\beta) = \int_r^1 (1 - \Phi) \int_r^1 (s - r) \frac{1}{1 - r} ds \frac{1}{1 - r} dr
$$

B does not enter when $\Pi_B(\beta) \geq c$:

$$
\Pi_B(\beta) = (1 - \Phi) \int_r^1 \frac{1}{1 - r} \left( \int_r^1 (s - r) ds \right) \frac{1}{1 - r} dr
$$

which defines the threshold equal to:

$$
\tau \geq 1 - \frac{4c}{1 - \Phi}
$$

with $\tau \in [0, 1]$ when $c < \frac{(1 - \Phi)}{4}$. Given that $\Pi_B(\beta)$ is everywhere decreasing in $\tau \in [0, 1]$, $\tau$ is the minimum threshold that preempts competition.

(ii) Let us first determine $R$’s expected profit offering a first bid $\beta_1' = 0$. In such case $B$ enters with probability 1 ($p = 1$) and from (3) $R$’s expected profit becomes

$$
\Pi_R(0; r, \omega_1) = \Phi \left[ H_0 r + (1 - H_0) \left( \int_0^r (r - s) dF(s) \right) \right]
$$

linear in $r \in [0, 1]$. This guarantees that the single-crossing conditions formalized by equations (12) and (13) are satisfied.

Given the threshold $\tau$ we can now characterize the minimum bid that deters $B$’s entry by requiring that the expected profit of raider with synergy $\tau$ offering a zero initial bid, given by (34), be (weakly) lower than $R$’s profit from preempting $B$’s entry i.e. $\tau - \beta$. Solving

$$
\Phi \frac{\tau}{2} (1 + H_0) \leq \tau - \beta
$$

in $\beta$ we find that

$$
\beta = \Phi \left( 1 - (1 + H_0) \frac{\Phi}{2} \right) = \left( 1 - \frac{4c}{1 - \Phi} \right) \left( 1 - (1 + H_0) \frac{\Phi}{2} \right)
$$

by substitution of (33).
As in Fishman (1988), the uniqueness of this equilibrium can be proved applying the credibility requirement of Grossman and Perry (1986).

In order to complete our proof we are left to find $I$’s expected payoff when \( \beta_1 = 0 \) which from (4) is equal to:

\[
\Pi_I(0; r) = \Phi(1 - H_0) \left( \int_0^r sdF(s) \right) + (1 - \Phi) \left( r + \frac{P}{\gamma} \right)
\]

\(
= (1 - \Phi) \left( r + \frac{P}{\gamma} \right) + (1 - H_0) \frac{P}{2} \tag{36}
\)

Finally, we have that

\[
\Pi_I(0; r) + \Pi_R(0; r) = (1 - \Phi) \left( r + \frac{P}{\gamma} \right) + (1 - H_0) \frac{P}{2} + (1 + H_0) \Phi r = r
\]

In the following, whenever \( \pi_R \) and \( \pi_I \) are written without specifying the range of \( r \), then results hold for any \( r \), i.e. \( r \leq \bar{r} \) and \( r \geq \bar{r} \) (respectively the case with preemption and without preemption).

**Proof of Lemma 1:** Suppose the game reaches the last stage \( T \) and \( S^T = \{\omega_l\} \). \( R \) can credibly threaten to opt-out by launching a tender offer and obtaining \( \pi_R - c_{to} > 0 \) by Assumption 1. Thus, in order to obtain an agreement with \( R \), \( I \) cannot claim more than \( r - (\pi_R - c_{to}) \). An agreement can be signed at bid \( \beta^T(\omega_l) = r - (\pi_R - c_{to}) \) and, given that \( \pi_R + \pi_I - c_{to} \leq r \) under Assumption 1, \( I \) is also better off obtaining \( \beta^T(\omega_l) \) than opting out and getting \( \pi_I \).\(^{27}\)

The only credible threat for \( R \) at state \( S^T = \{\omega_h\} \) is to exit without launching a tender offer, getting a payoff equal to zero. If \( R \) exits at \( S^T = \{\omega_h\} \), then \( I \) keeps the control of the target firm and the private benefits \( P/\gamma \). If \( r > P/\gamma \) an agreement is possible at any bid \( \beta^T(\omega_h) \) such that:

\[
\frac{P}{\gamma} \leq \beta^T(\omega_h) \leq r
\]

so that \( I \) asks for a minimum bid \( \beta^T(\omega_h) = r \) to \( R \) who agrees to sign the deal at this offer. If otherwise \( r < P/\gamma \), \( R \) simply exits and \( I \) keeps the private benefits \( P/\gamma \). In any case \( R \) obtains a payoff of zero, while the payoff for \( I \) is equal to \( \max \{r, P/\gamma\} \).

Going backward we get to the analysis of the bargaining stage \( T - 1 \) given the continuation payoffs. Let us start with \( S^{T-1} = \{\omega_h\} \). \( R \) given (10), players know that their continuation payoffs are equal to (21) when \( r > P/\gamma \). Thus, if

\[^{27}\text{Indeed, we have:}
\]

\[
\beta^T(\omega_l) = r - (\pi_R - c_{to}) > \pi_I
\]

\[
\pi_R + \pi_I - c_{to} < r
\]

\[
c_{to} > r - (\pi_R + \pi_I)
\]

that is true by Assumption 1.
$R$ is called to play at the last node of stage $T-1$, then she will choose to stay in the game$^{28}$; anticipating this decision, at the previous node $I$ prefers to call a private auction, because in this way he obtains a payoff $r+P/\gamma - \tau > r$ when $\tau < P/\gamma$. Hence, given that $I$'s threat to go to a private auction is credible, an agreement is not profitable for $R$ because it would require offering $I$ more than $r$. Consequently, the unique equilibrium at $S^{T-1} = \{\omega_h\}$ is:

$$\sigma^{T-1}_R(\omega_h) = \{0; \text{stay}\}$$
$$\sigma^{T-1}_I(\omega_h) = \{PA\}$$

and the equilibrium payoffs are equal to (23).

In the case $r < P/\gamma$ instead, $R$ knows the best she can obtain from the negotiation is zero, so she can leave the negotiation without calling for a tender offer. Anticipating this, $I$ can call a private auction obtaining $r + P/\gamma - \tau$. If $I$ rejects and delays the deal, the game moves to $S^T = \{\omega_h\}$ where $I$ gets $P/\gamma$. With $r < P/\gamma$, we have that $r + P/\gamma - \tau < P/\gamma$ with $\tau > r$. Thus at equilibrium, with $r < P/\gamma$ and $\tau \in [r, P/\gamma]$ no deal can be reached and $R$ exits with zero, while $I$ keeps the private benefits $P/\gamma$. With $r < P/\gamma$ and $\tau < r$ we have at $S^{T-1} = \{\omega_h\}$ the same equilibrium as in the case $r > P/\gamma$.

If the game is in $S^{T-1} = \{\omega_I\}$, when $R$ is called to play, she has two options: to opt out, launching a tender offer and obtaining $\pi_R - c_{to} > 0$; or to stay in the negotiation. In this latter case her expected payoff is equal to $e^{-\lambda}(\pi_R - c_{to}) < \pi_R - c_{to}$ by (7) and (8). Thus, $R$ goes to the tender offer. Anticipating this decision, the credible threat for $I$ at $S^{T-1} = \{\omega_I\}$ is the following: $I$ can call a private auction at his turn before $R$ obtaining a payoff $\pi_I - \tau$; or he can reject the offer and delay ending up with $\pi_I$, since $I$ rationally anticipates that $R$ chooses a tender offer. In order to obtain an agreement, $R$ then has to offer at least $\beta^{T-1}(\omega_I) = \pi_I$ to $I$. Under Assumption 1, $R$ prefers to secure a deal rather than to launch a tender offer, because $r - \beta^{T-1}(\omega_I) = r - \pi_I \geq \pi_R - c_{to}$.

The unique equilibrium at $S^{T-1} = \{\omega_I\}$ is then:

$$\sigma^{T-1}_R(\omega_I) = \{\beta^{T-1}(\omega_I), \text{TO}\}$$
$$\sigma^{T-1}_I(\omega_I) = \{\text{agree}\}$$

which in turn defines the equilibrium payoffs at the beginning of stage $T-1$ as stated in the lemma.

**Proof of Lemma 2:** (i) $r > P/\gamma$ or $r < P/\gamma$ and $\tau < r$.

At $S^{T-1} = \{\omega_I\}$ Lemma 1 shows that $R$ can credibly threaten to opt out if $I$ rejects $R$’s offer and stays into the negotiation, because $e^{-\lambda}(\pi_R - c_{to}) < \pi_R - c_{to}$. Recall that $x^t$ is the expected payoff for $R$ in state $S^t = \{\omega_I\}$, $t < T-1$, given that the game proceeds to stage $t+1$ (that is, if no agreement is reached and no player opts out at $S^t = \{\omega_I\}$). Let us start studying the case $r > P/\gamma$: when $t = T-2$, we have:

$$x^{T-2} = e^{-\lambda}(r - \beta^{T-1}(\omega_I)) + (1 - e^{-\lambda})\tau$$

$^{28}$Opting out, $R$ gets $-c_{to}$, while ending he gets zero for sure.
Therefore if no deal is signed at $T - 2$ and the game proceeds to stage $T - 1$, then the continuation payoffs for $R$ are given by (22) and (23). With an abuse of notation let $x^{T-1} = r - \beta^{T-1}(\omega_i)$.\(^{29}\) Assuming that no agreement is reached and no player opts out before $S^{T-1} = \{\omega_i\}$ as long as the state of the game stays at $\omega_i$, we can repeat the same argument for any $S^t = \{\omega_i\}$, $t \in [1, T - 1]$, obtaining:

$$x^{t-1} = e^{-\lambda}x_t + (1 - e^{-\lambda})\tau$$

Solving recursively the above equation, where for clarity we replace the time to the end $(T - 1) - t$ with variable $\chi$, we have:

$$x^t = e^{-\lambda\chi} \left( r - \beta^{T-1}(\omega_i) \right) + (1 - e^{-\lambda\chi})\tau$$  \hspace{1cm} (37)

If $r - \beta^{T-1}(\omega_i) > \tau$ (37) implies that $x^t$ increases with $t$. On the contrary, if $r - \beta^{T-1}(\omega_i) < \tau$, $x^t$ is decreasing in $t$. We now need to distinguish two cases (a) $r \geq \tau$ and (b) $r < \tau$. The condition $r - \beta^{T-1}(\omega_i) > \tau$ leads to the following restrictions on the value of $\tau$ in the two cases:

(a) $r - \beta^{T-1}(\omega_i) = r - \beta > \tau \Leftrightarrow r - \tau \left( 1 - (1 + H_0) \frac{\Phi}{\gamma} \right)$; and

(b) $r - \beta^{T-1}(\omega_i) = r - \pi_I > \tau \Leftrightarrow r - \left( (1 - \Phi) \left( r + \frac{\chi}{\gamma} \right) + \Phi (1 - H_0) \frac{\tau}{\gamma} \right) > \tau$

which is equivalent to:

$$\tau < \Phi (1 + H_0) \frac{\tau}{\gamma} - (1 - \Phi) \frac{P}{\gamma}$$

Summarizing, if $r > P/\gamma$ and $x^{(T-1)-\chi} = e^{-\lambda\chi} \left( r - \beta^{T-1}(\omega_i) \right) + (1 - e^{-\lambda\chi})\tau$, $x^t$ is increasing in $t$ only if termination fees are sufficiently low, i.e. $\tau < \min \left\{ r - \tau (1 - (1 + H_0) \frac{\Phi}{\gamma}); \Phi (1 + H_0) \frac{\tau}{\gamma} - (1 - \Phi) \frac{P}{\gamma} \right\}$ which determines our assumption (A2).

Notice also that at $\chi = 0$ (i.e. at $t = T - 1$) then $x^{T-1} = r - \pi_I > \pi_R - c_{to}$ by Lemma 1 irrespective of the value of the synergy $r$.

Similarly, recall that $y^t$ is $I'$s expected payoff in state $S^t = \{\omega_i\}$ when the game proceeds to stage $t + 1$ (i.e. no agreement is reached and no player opts out at $S^t = \{\omega_i\}$). By Lemma 1, at $S^{T-1} = \{\omega_i\}$ a deal is signed at equilibrium at the offer $\beta^{T-1}(\omega_i) = \pi_I$. If no deal is signed at $T - 2$ and $r > P/\gamma$ then the continuation payoffs for $I$ at $T - 1$ are given by (22) and (23) and we can write:\(^{30}\)

$$y^{T-2} = e^{-\lambda} \beta^{T-1}(\omega_i) + (1 - e^{-\lambda}) \left( r + \frac{\chi}{\gamma} - \tau \right)$$

As before, assuming that (i) no agreement is reached at any stage $t < T - 1$, no player opts out before $S^{T-1} = \{\omega_i\}$, and (ii) that the state of the game

\(^{29}\)It is commonly known that at the last stage an agreement is certainly reached at $S^{T-1} = \{\omega_i\}$: hence, the game does not continue at $S^{T-1} = \{\omega_i\}$.

\(^{30}\)Again, $y^{T-1} = b^{T-1}(\omega_i)$ with an abuse of notation because the game ends at $T - 1$ at equilibrium.
stays at \( \omega_t \), the argument can be repeated backward for any \( S^t = \{ \omega_t \} \) with \( t \in [1, T - 1] \). We then obtain:

\[
y^{t-1} = e^{-\lambda} y^t + (1 - e^{-\lambda}) (r + \frac{p}{\tau} - \tau)
\]

Solving this recursive equation:

\[
y^t = e^{-\lambda \gamma} \beta^{T-1}(\omega_t) + (1 - e^{-\lambda \gamma}) \left( r + \frac{p}{\tau} - \tau \right)
\]

and \( y^t \) decreases with \( t \) reaching a minimum at \( y^{T-1} = \beta^{T-1}(\omega_t) \) if \( r + \frac{p}{\tau} - \tau > \beta^{T-1}(\omega_t) \). If we are in case (a) where \( r \geq \tau \) and \( \beta^{T-1}(\omega_t) = \beta \), then the condition \( r + \frac{p}{\tau} - \tau > \beta \) is trivially satisfied under the conditions on \( \tau \) determined above.

The same holds if we are in case (b) where \( r < \tau \) and \( \beta^{T-1}(\omega_t) = \pi_I \) as in this case it must be \( r + \frac{p}{\tau} - \tau > \pi_I \) which again is guaranteed by (A2).

(ii) \( r < \frac{p}{\tau} \) and \( \tau \in [r, \frac{p}{\tau}] \),

Lemma 1 shows that \( R \) obtains a zero payoff in \( \{ \omega_h \} \), hence \( x^{(T-1)-\gamma} = e^{-\lambda \gamma} \left( r - \beta^{T-1}(\omega_t) \right) \), which is increasing in \( t \) if \( r - \beta^{T-1}(\omega_t) > 0 \) i.e.

(a) \( r \geq \tau : r - \beta^{T-1}(\omega_t) = \beta - \beta > 0 \)

(b) \( r < \tau : r - \beta^{T-1}(\omega_t) = r - \pi_I > 0 \)

which are both verified by (A2) as \( r - \beta > \tau \geq 0 \) and \( r - \pi_I > \tau \geq 0 \). \( I \)'s continuation payoff at \( T - 1 \) is equal to \( \frac{p}{\tau} \) by Lemma 1 and

\[
y^{(T-1)-\gamma} = e^{-\lambda \gamma} \beta^{T-1}(\omega_t) + (1 - e^{-\lambda \gamma}) \frac{p}{\tau}
\]

which is decreasing in \( t \) if \( \frac{p}{\tau} - \beta^{T-1}(\omega_t) > 0 \). Again distinguishing the two cases:

(a) \( r \geq \tau : \beta^{T-1}(\omega_t) = \beta \) and \( \frac{p}{\tau} - \beta > 0 \) since \( \beta < r < \frac{p}{\tau} \);

(b) \( r < \tau : \beta^{T-1}(\omega_t) = \pi_I \) and \( \frac{p}{\tau} - \pi_I > 0 \) if \( \Phi \frac{p}{\tau} - r \left( (1 - \Phi) + (1 - H_0) \frac{p}{\tau} \right) > 0 \) which is true for sufficiently low \( r \).

\textbf{Proof of Proposition 2:} The proof of the proposition makes use of the following lemma.

\textbf{Lemma 3:} Let \( t < T - 2 \) be an odd number so that \( R \) makes the first offer, and assume that A1 and A2 are verified, \( r > P/\gamma \) and \( r > \pi_I \). Then the unique PBE of the game in state \( S^t = \{ \omega_t \} \) is to reach an agreement at offer \( \beta^t(\omega_t) = \pi_I \) if either \( x^{t+1} < \pi_R - c_{o\sigma} \) or \( x^{t} < \pi_R - c_{o\sigma} < x^{t+1} \). Otherwise, if \( \pi_R - c_{o\sigma} < x^{t} < x^{t+1} \) then, for any odd \( t < T - 2 \), at \( S^t = \{ \omega_t \} \) the game continues to stage \( t + 1 \).

\textbf{Proof of Lemma 3:} Consider the case when \( x^{t+1} < \pi_R - c_{o\sigma} \) and let \( t \) be an odd number. At state \( S^{t+1} = \{ \omega_t \} \) \( I \) starts making an offer and, because \( x^{t+1} < \pi_R - c_{o\sigma} \), \( R \) can credibly threaten to opt-out at \( t + 1 \). So, for \( R \) to accept any offer by \( I \), \( I \) must match at least \( R \)'s opt-out payoff \( \pi_R - c_{o\sigma} \). At such an offer, \( R \) agrees and leaves \( I \) with a payoff \( r - (\pi_R - c_{o\sigma}) > \pi_I \) the payment \( I \)
would get when \( R \) opts out. Hence, at equilibrium an agreement is reached at \( t + 1 \) with the following continuation payoffs:

\[
\Pi^{t+1}_R(\omega_l) = \pi_R - c_{lo} \\
\Pi^{t+1}_I(\omega_l) = r - (\pi_R - c_{lo}) > \pi_I
\]

We use the previous result to analyze what happens at stage \( t \), when \( R \) starts making an offer. By Lemma 2 we have \( x^t < x^{t+1} < \pi_R - c_{lo} \), hence opting-out is also a credible threat for \( R \) at \( S^t = \{\omega_l\} \). Because of that, \( R \) can induce \( I \) to accept an agreement payoff \( \pi_I \). \( I \) rationally anticipates this and accepts any offer \( \beta(\omega_l) \geq \pi_I \), with this condition holding as an equality at the equilibrium.

Consider now the case \( x^t < \pi_R - c_{lo} < x^{t+1} \). We start again by analyzing the equilibrium at \( S^{t+1} = \{\omega_l\} \), when \( I \) makes an offer. By Lemma 2, if \( I \) is called to play at the last node of this stage, \( I \) always prefers to stay rather than to opt out because \( y^{t+1} > \pi_I, \forall t \). Knowing this, \( R \)'s payoff from rejecting the offer and delaying is equal to \( x^{t+1} > \pi_R - c_{lo} \), hence \( R \) does not opt-out at \( t + 1 \). To induce \( R \) to agree, \( I \) has to offer \( R \) at least \( x^{t+1} \), and in such a case the payoff for \( I \) would be at most \( r - x^{t+1} \). By Lemma 2, \( y^{t+1} > r - x^{t+1} \) that implies that, if \( S^{t+1} = \{\omega_l\} \), then \( I \) prefers to continue the negotiations rather than to reach an agreement with \( R \). Thus, at \( S^t = \{\omega_l\} \) both players know that at state \( S^{t+1} = \{\omega_l\} \) there will not be an agreement at equilibrium when \( x^{t+1} > \pi_R - c_{lo} \).

Let us now move backward to \( S^t = \{\omega_l\} \). Given that there is no deal at equilibrium at \( S^{t+1} = \{\omega_l\} \), \( x^t \) represents the continuation payoff for \( R \) to stay in the game if the last node of stage \( t \) is reached, and because \( x^t < \pi_R - c_{lo} \), \( R \) opts out at this point. Anticipating this, \( I \) prefers to reject and delay any unsatisfying offer rather than to opt out thereby avoiding to pay the termination fees \( \tau \). Thus, in order to induce \( I \) to agree, \( R \) must offer at least \( \beta(\omega_l) = \pi_I \) that leaves \( R \) with a payoff at most equal to \( r - \pi_I \). Any offer lower than \( \pi_I \) would induce \( I \) to stay in the negotiation without an agreement, and this would in turn induce \( R \) to opt-out because \( x^t < \pi_R - c_{lo} \). In conclusion, if a deal is not agreed, then, by Assumption 1, \( R \) gets at most \( \pi_R - c_{lo} < r - \pi_I \). Hence, \( R \) is better off offering \( \beta(\omega_l) = \pi_I \) and signing a deal with \( I \). This outcome is supported by the fact that \( R \)'s opt-out threat is credible.

The last case is \( \pi_R - c_{lo} < x^t < x^{t+1} \). Suppose that at \( S^t = \{\omega_l\} \), \( R \) makes the first offer. By the previous argument, there is no deal at equilibrium at \( t + 1 \), so when the last decision node of stage \( t \) that belongs to \( R \) is reached, then \( R \) prefers to stay in the game because opting out is not a credible threat \( \beta(\omega_l) = x^t \). Hence, opting out is not a credible threat for \( I \) either. Thus, when making the first offer, \( R \) knows that \( I \) will sign a deal only at an offer at least equal to \( \beta(\omega_l) = y^t \). But by doing so, \( R \) obtains a payoff smaller (or equal) than \( r - y^t \); thus \( R \) cannot credibly threaten to opt out at the last node of stage \( t \), and, so, her outside option payment is \( x^t \). But by Lemma 2, (26): \( x^t + y^t > r \Leftrightarrow r - y^t < x^t \). Hence for \( R \) it is always optimal not to propose an agreement.
and stay in the negotiation. $I$ plays the same strategy so at equilibrium the negotiation proceeds to stage $t+1$. □

To conclude the proof of Proposition 1, note the following. By Lemma 1, opting out is a credible threat for $R$ at $T-1$: indeed, by Assumption 1, $R$’s payoff $R$ at $S^{T-1}(\omega_I) = r - \beta^{T-1}(\omega_I) = r - \pi_I > \pi_R - c_{to}$. More generally by Lemma 3 we have that opting out is a credible threat as soon as $x^t < \pi_R - c_{to}$ or $x^{t+1} < \pi_R - c_{to}$, and, when one of these two inequalities holds a deal is signed at bid equal to $\pi_I$. Thus, if an (odd) $t < T-1$ s.t. $x^{t+1} < \pi_R - c_{to}$ exists, then there will be a unique equilibrium where a deal is signed at bid $\beta^t(\omega_I) = \pi_I$.

But if this is the case, then, by Lemma 2, also $x^t < x^{t+1} < \pi_R - c_{to}$, hence an agreement is signed in the previous stage $t$ (odd, so $R$ makes the first offer).

This proves the first part of the Proposition.

Because $x_t$ is increasing with $t$ under the stated assumptions (see Lemma 2), when $x^1 > \pi_R - c_{to}$ the $R$’s opt-out threat is never credible until $T - 1$ provided the game stays in state $\omega_I$. Thus the game continues as long as the state is $\omega_I$ until stage $T - 1$, when an agreement is signed at $\beta^{T-1}(\omega_I) = \pi_I$ (Lemma 1).

Further, to compute the players expected payoffs when $x^1 > \pi_R - c_{to}$ notice that with $r > \frac{P}{\gamma}$, $R$ (resp. $I$) earns $\tau$ (resp. $r + \frac{\tau}{\gamma} - \tau$) as soon as the game falls into state $\omega_h$, while $R$ ($I$) earns $\pi_R - c_{to}$ (resp. $\pi_I$) if $S^{T-1} = \{\omega_I\}$, an event with probability $e^{-\lambda(T-1)}$. Thus $R$ obtains

$$
\tau(1-e^{-\lambda}) \left( \sum_{j=0}^{T-1} e^{-\lambda j} \right) + (\pi_R - c_{to})e^{-\lambda(T-1)} = \tau(1-e^{-\lambda}) \frac{1 - e^{-\lambda(T-1)}}{1 - e^{-\lambda}} + (\pi_R - c_{to})e^{-\lambda(T-1)}
$$

which provides the expression in the text of the proposition. Analogous procedure gives the expected payoff for $I$. The expressions for the expected payoffs in case $r < \frac{P}{\gamma}$ are obtained analogously considering the equilibrium actions of $R$ and $I$ at $\omega_h$ described in Lemma 1. ■

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